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Observational limits on a millihertz stochastic background of gravitational radiation

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OBSERVATIONAL LIMITS ON A MILLIHERTZ STOCHASTIC BACKGROUND OF GRAVITATIONAL RADIATION

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ABSTRACT

One hundred station years of data collected by the Project IDA global network of seismometers have been analyzed to look for anomalous excitation of the low-frequency quadrupole modes of Earth. The 1 σ upper limit of the average vertical ground motion at 0.31 mHz, the frequency of the fundamental quadrupole mode, is 3.4×10^{-7} m. This implies an upper limit of the gravitational wave energy spectral density at this frequency of 1.7×10^{-5} J m^{-3} Hz which corresponds to about three times the closure density of the universe per octave. During periods of low earthquake activity, the 1 σ upper limit at 1.72 mHz, the frequency of the fourth harmonic, is 8.5×10^{-9} m. The corresponding limit of the gravitational wave energy density at this frequency is 6.1×10^{-6} J m^{-3} Hz or about six times the closure density per octave. The ultimate sensitivity of Earth as a gravitational radiation detector is discussed.

Subject headings: cosmology — gravitation — radiation mechanisms

I. INTRODUCTION

Previous studies of the seismic effects on Earth of a stochastic background of gravitational waves at millihertz frequencies (Boughn and Kuhn 1984, and references cited therein) have led to upper limits of the energy spectral density of such a background which correspond to 4×10^3 and 2×10^2 times the closure density of the universe per octave at 0.31 and 1.72 mHz, respectively. Closure density is a benchmark since energy densities much higher than this value can be ruled out on cosmologial grounds. These limits were primarily due to small data sets and inadequate spectral resolution of published seismic data. The study described in this paper analyzed more than 100 station years of data of the Project IDA (International Deployment of Accelerometers) global digital seismic network (Agnew *et al.* 1986) with frequency resolution adequate to resolve the high Q, fundamental quadrupole mode of Earth.

The use of Earth as a gravitational wave (GW) detector was first discussed by Weber and his collaborators nearly 30 years ago (Foward *et al.* 1961). Current spherically symmetric Earth models (Gilbert and Dziewenski 1975) are accurate enough to calculate the interaction cross section of Earth with GWs to within a few percent. Only the poloidal Earth modes associated with spherical harmonics of order $l = 2$, i.e., $Y_2{}^m$, have the proper quadrupole symmetry to couple to GWs. Poloidal modes are designated by ${}_nS_{lm}$ where n represents the radial eigenvalue (Aki and Richards 1980). For a spherically symmetric system, the frequencies of these modes are independent of m (the standard $2l + 1$ degeneracy); consequently, the m subscript is usually suppressed. Using the equation of geodesic deviation, Boughn and Kuhn (1984) derived the following expression for the response of an accelerometer on Earth's surface to an isotropic background of GWs

$$\langle x^2 \rangle = \frac{8\pi^2 G}{75c^2} R_E{}^2 \tau_n K_n{}^2 I_n{}^2 S(\nu_n) , \qquad (1)$$

where $\langle x^2 \rangle$ is the mean square vertical displacement of Earth's surface, G is Newton's constant, c is the speed of light, R_E is the radius of Earth, K_n is a numerical constant on the order of unity which is related to accelerometer response, ν_n is the frequency of the nth mode, τ_n is the amplitude decay time, and $S(\nu)$ is the spectral energy density of the GW background. I_n is related to the overlap integral of the nth order quadrupole eigenfunction with the force density profile of the GW. The fundamental ($n = 0$) is the most sensitive to GWs ($I_0 = 0.7$) while higher n modes couple less strongly because the radial structure of the eigenfunctions does not match that of the gravitational force. The strongest coupling harmonic is the fourth ($I_4 = 0.087$).

II. ANALYSIS OF IDA DATA

The IDA global network (Agnew *et al.* 1986) consists of 23 strategically placed accelerometers which are intended to monitor Earth normal mode vibrations from the fundamental quadrupole mode with a period of 54 minutes to higher harmonic and multipole modes with frequencies up to 50 mHz. Since it began operating in 1975 nearly 200 station years of data have been collected. Figure 1 is the weighted average of 1165 "one-month" (2,621,440 s) spectra in the frequency range 0.25–3.0 mHz. The time duration was chosen so as to barely resolve the width of the ${}_0S_2$ mode. The prominent peaks correspond to the fundamental modes of various multipole order, i.e., ${}_0S_l$, as indicated in the figure. These modes are easily excited by earthquakes. Because $l \neq 2$ modes cannot be excited by GWs they prove useful in evaluating the terrestrial excitation of the quadrupole modes as will be discussed below.

The broad-band background in this spectrum is apparently due to barometric pressure fluctuations (Murphy and Savino 1975), and it is this source which constitutes the background noise. If the underlying source is a Gaussian process, then a sample of power spectra at any given frequency will have an exponential probability distribution with a variance equal to the square of the mean power (Groth 1975). Averaging N such spectra results in a variance equal to $1/N$ times the square of

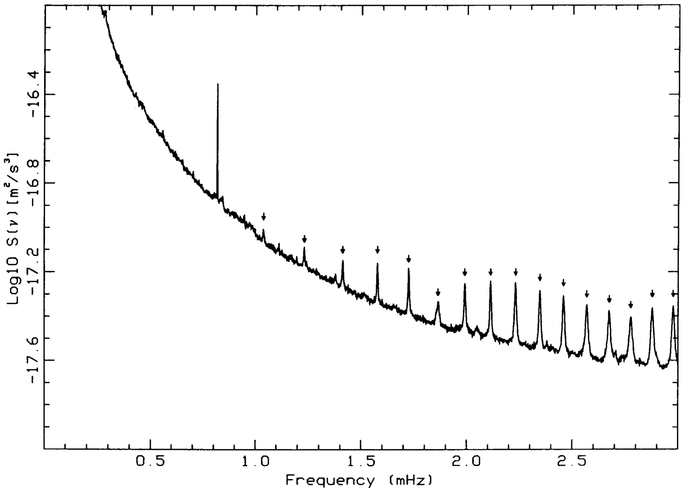

FIG. 1.—Average vertical acceleration power spectrum for 100 station years of IDA data. The arrows indicate the ${}_0S_l$ modes for $6 \le l \le 21$. The large feature at 0.81 mHz corresponds to the ${}_0S_0$ high-Q, scalar mode.

the mean power. The ratio of the variance to mean power squared for the average spectrum in Figure 1 is on the order of 800, which is somewhat smaller than the number (1165) of spectra averaged. This is due to the fact that spectra with larger broad-band power were given less weight as will be discussed below.

a) Cleaning Procedure

Data were processed in blocks of 2^{17} points sampled at 20 s intervals. An eight pole high pass filter was applied in the frequency domain to each block to remove the 12 hr tide signal which otherwise dominates the seismometer output. The data were then examined in subblocks of 1024 points which overlapped by 304 points. If the total power in the 5–20 mHz range in these subblocks exceeded a threshhold value, it was flagged as "earthquake noisy" and eliminated from the data. This procedure eliminated most instrumental noise spikes as well. The 850 data points at the beginning of each block were also eliminated because of ringing of the high pass filter as were the 850 points following a tape record gap if the gap exceeded 33 minutes. Finally, two successive 3 σ cuts were applied to the data to eliminate the occasional isolated instrumental noise spike. If more than 30% of the points were edited from any data block it was removed from further consideration. The fraction of data eliminated was accounted for in the weighted averages (see below) and did not, therefore, bias the result.

b) Incoherent Average of Spectra

In order to maximize the signal to noise it is clear that the spectra of stations with larger broadband power should be given less weight. The Neyman-Pearson lemma proves that the "likelihood ratio" is the best statistic in the sense of maximizing the probability of detection (Whalen 1971). In the typical situation of searching for a signal in the presence of Gaussian noise this reduces to the usual weighted least-squares method (see, for example, Bevington 1969). In the present context of searching for a small, random signal (GW's) in the presence of a larger random signal (barometric fluctuations) the likelihood ratio implies that if the spectra are statistically independent they should be weighted inversely with the square of the mean power (Boughn, Saulson, and Uson 1986), i.e.,

$$\langle S(\nu) \rangle = \frac{\sum_k [S_k(\nu)\beta_k^2/B_k^2]}{\sum_k (\beta_k^4/B_k^2)}, \tag{2}$$

where $S_k(\nu)$ is the kth power spectral density of ground displacement at frequency ν, B_k is the average broad-band power for the kth spectra, and β_k is the fraction of the data which survives the editing procedure. The average spectrum of Figure 1 was evaluated in four frequency intervals: 0.25–0.5 mHz, 0.5–1.0 mHz, 1.0–2.0 mHz, and 2.0–3.0 mHz. The B_k were computed separately for each interval. The additional weighting by β_k is included to take into account the fraction of the data deleted by the above cleaning procedure. The peak power spectral density of a narrow-band feature, e.g., Earth mode, is reduced by two factors of β_k, one factor from the average decrease of power of that spectra, and the other factor is due to the spread of the power out into distant sidelobes where it is indistinguishable from the background. The fourth power of β_k in the denominator is needed to provide an unbiased estimate of the strength of a spectral feature. Figure 1 represents the incoherent average of 1165 spectra.

The assumption that the spectra are statistically independent is not entirely accurate. In any given month the hypothetical GW signals detected by the IDA stations in any given ${}_0S_{lm}$ mode will be highly correlated. Even though the $2l + 1$ degeneracy associated with m is removed by the rotation of Earth, the frequency splitting is usually small compared to the finite width of the resonance curves and is, therefore, unresolved in most modes (Dahlen and Smith 1975). For this reason the correlations of signals of the different stations remain hidden and the best one can do is the incoherent average expressed in equation (2). This is not the case for the ${}_0S_{2m}$ modes which are highly resolved, and the signals from individual stations can be coherently averaged as is discussed below.

c) Coherent Average of Spectra

Since the five ${}_0S_2$ modes are highly resolved (see Table 1), each mode can be treated independently. The angular eigenfunctions of these modes are proportional to $Y_{lm}(\theta, \Phi)e^{i\omega_m t}$. It is clear that $m \neq 0$ modes correspond to traveling waves of angular velocity $-\omega_m/m$ in the azimuthal direction, and the responses of the various stations can be coherently stacked by simply translating a given station's response by a time = $[(-m/\omega_m) \times$ station longitude$]$ before averaging. An equivalent operation in the frequency domain is to phase shift the Fourier transform, $x(\omega)$, of the signal by $(m\Phi_k)$ where Φ_k is the longitude of the kth station.

If the underlying background noise is Gaussian, then both the real and imaginary parts of the Fourier transform will have a Gaussian probability distribution with a variance equal to one-half the mean power (Groth 1975), and the usual weighted least-squares procedure (Bevington 1969) indicates the individ-

TABLE 1

FREQUENCIES OF ${}_0S_2$ MODES

m	ν_m (mHz)
+2	0.2999
+1	0.3047
0	0.3094
−1	0.3140
−2	0.3186

ual Fourier transforms should be weighted inversely with respect to the background power. Of course, the transforms must also be weighted with the expected lattitude dependence of the signal and β_k which represents the fraction of the data which survived the cleaning procedure. Therefore, the Fourier transforms of the signals from each station for the ith month and azimuthal eigenvalue, m, were averaged according to

$$\langle x(\omega)\rangle_{i,m} = \frac{\sum_k x_k(\omega) f_m(\theta_k) e^{im\Phi_k} \beta_k / B_k}{\sum_k f_m^2(\theta_k) \beta_k^2 / B_k}, \quad (3)$$

where k = IDA station, $x(\omega)$ = Fourier transform of seismometer response, $f_m(\theta)$ = the latitude dependence of the mth eigenfunction, θ = latitude, Φ = longitude, B = broad-band power, and β = fraction of data analyzed. The denominator is that required to give an unbiased estimate of the amplitude of the mth mode. It is also easily shown that the denominator is the inverse of the variance of $\langle x(\omega)\rangle_{i,m}$ i.e.,

$$\frac{1}{\sigma_{i,m}^2} = \sum_k f_m^2(\theta_k) \beta_k^2 / B_k . \quad (4)$$

The standard relationship between the power spectrum and finite Fourier transform is $S(\nu) = 2S(\omega) = (2/T)|x(\omega)|^2$, where T is length of the data sample.

If the signals from different months are assumed to be statistically independent then, as before, the power spectra of each month, $S_{i,m}(\nu)$, should be weighted inversely with the square of the variance before averaging, i.e.,

$$\langle S(\nu)\rangle_m = \frac{\sum_i S_{i,m}(\nu)/\sigma_{i,m}^4}{\sum_i 1/\sigma_{i,m}^4} . \quad (5)$$

Because of the normalization chosen for the modes, the power spectrum of vertical ground motion is the above power spectrum for the normal mode amplitude divided by 4π. The resultant average power spectra for all five ${}_0S_2$ modes are depicted in Figure 2. These spectra represent the weighted average of ~100 monthly power spectra, each of which was obtained by coherently averaging the data from typically 12 IDA stations.

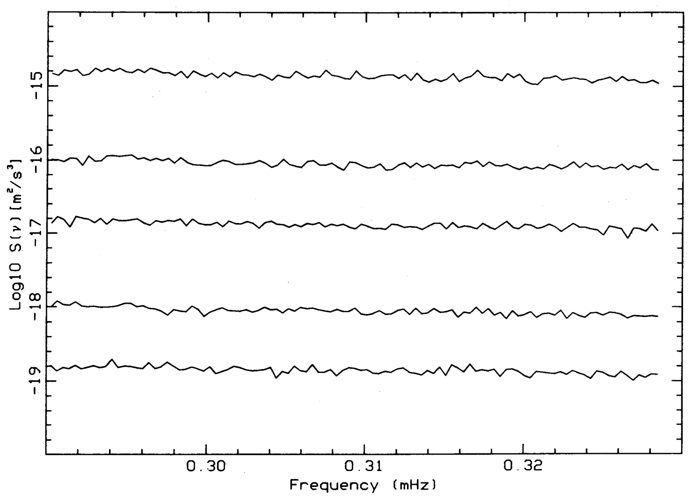

FIG. 2.—Average acceleration power spectra in the region of the ${}_0S_2$ modes. The spectra correspond from top to bottom to the $m = +2, +1, 0, -1, -2$ modes. The spectra have been displaced by $\Delta[\log S(\nu)] = m$ for plotting. Note the $m = \pm 1$ modes have slightly lower power than the others.

III. EARTHQUAKE EXCITATION OF EARTH MODES

As mentioned above, the prominent peaks in Figure 1 correspond to the fundamental poloidal modes of different harmonic order, l, and the average power in these modes is presumably due to earthquakes. In order to check the reliability of both the data analysis procedure and the IDA data itself as well as to estimate the expected contamination of the ${}_0S_2$ mode by earthquakes, the average excitation by earthquakes for the year 1984 was calculated. It is generally accepted that the "slip-fault" or "double-couple" model provides a good description of most earthquakes (Aki and Richards 1980). Furthermore for low-frequency modes, earthquakes can be assumed to be point sources that occur instantaneously. With these assumptions, the vertical ground displacement of a particular Earth mode is given by (Gilbert 1971)

$$x(\theta, \Phi, t) = \sum_{j,k} e_{jk}^* M_{jk} U_r(\theta, \Phi) \left\{ \frac{1 - e^{-t/\tau} \cos \omega t}{\omega^2} \right\}, \quad (6)$$

where e_{ij} = strain in the normal mode at the site of the earthquake; M_{ij} = seismic moment tensor of the earthquake; $U_r(\theta, \Phi)$ = the radial component of the eigenfunction at the longitude Φ, latitude θ, and surface of Earth; τ = amplitude decay time of the mode, and ω = angular frequency of the mode.

The root mean square (rms) surface accelerations at the locations of the IDA stations due to all earthquakes in 1984 which had seismic moments $\geq 10^{24}$ dyne cm (which corresponds roughly to 5.2 mag on the Richter scale) were computed for several ${}_0S_l$ modes. These were then averaged using the same weighting and windowing as for the actual IDA data. The results are listed in Table 2 along with the observed values determined from the corresponding spectral peaks of the 1984 IDA data. The higher frequencies have decay times of several hours, and the editing procedure described above reduces their strengths significantly. The errors listed are those associated with the formal fits of Lorentzian line shapes to the data. Estimates of the components of the seismic moment tensor and locations of the sources were taken from *The Preliminary Determination of Epicenters* (US Geological Survey 1984). For large earthquakes, the double-couple parameters listed in that publication were computed by two different inversion

TABLE 2

COMPARISON OF OBSERVED AND PREDICTED rms ACCELERATION FOR SEVERAL ${}_0S_l$ MODES

Mode	Observed (10^{-12} m s^{-2})	Computed (10^{-12} m s^{-2})
${}_0S_0$	2.2 ± 0.1	1.4
${}_0S_7$	2.8 ± 0.4	1.9
${}_0S_8$	3.5 ± 0.3	3.5
${}_0S_9$	3.4 ± 0.2	3.1
${}_0S_{10}$	4.0 ± 0.2	4.9
${}_0S_{11}$	4.6 ± 0.3	3.0
${}_0S_{12}$	4.4 ± 0.3	7.1
${}_0S_{13}$	4.2 ± 0.2	5.5
${}_0S_{14}$	5.7 ± 0.1	7.8
${}_0S_{15}$	4.9 ± 0.2	5.8
${}_0S_{16}$	5.8 ± 0.1	9.9
${}_0S_{17}$	5.8 ± 0.1	7.1
${}_0S_{18}$	5.3 ± 0.3	8.9
${}_0S_{19}$	6.1 ± 0.3	6.2
${}_0S_{20}$	6.5 ± 0.1	11.3
${}_0S_{21}$	6.0 ± 0.7	8.8

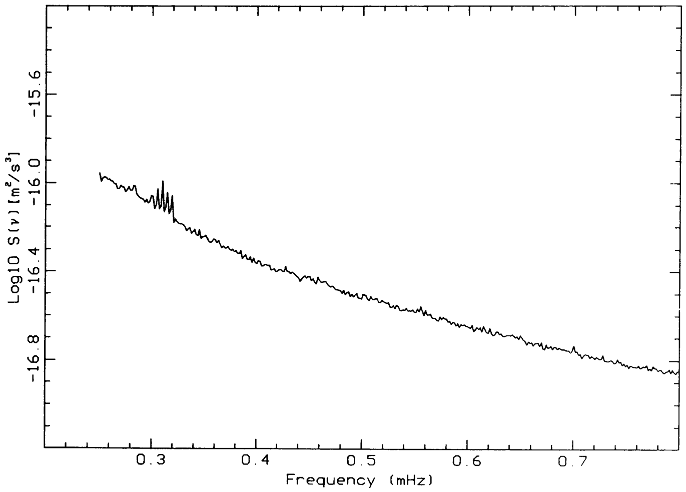

FIG. 3.—Same as Fig. 1 but with a fake signal included

methods, the "moment tensor solution" (Sipkin 1982) and the "centroid moment tensor" (Dziewonski, Chou, and Woodhouse 1981). Typically these two solutions agree to within less than a factor of 2. When both sets of parameters were listed, an average of the two was taken. Considering the inaccuracies in the determination of the seismic moments, the results of the calculations are consistent with the observations. The values listed in Table 2 are dominated by the large earthquakes: the strongest 15% of the earthquakes are responsible for more than 99% of the power in the ${}_0S_l$ modes.

It should be noted that no excitations of the ${}_0S_2$ modes are evident in either Figures 1 and 2, or in the 1984 data nor is any expected from earthquakes. Both of these points will be discussed below in the context of current limits on the gravitational wave background and the ultimate sensitivity of Earth as a gravitational wave detector. In order to check the detection algorithm for these modes, a single-pulse excitation of all five modes was added to the raw data of all IDA stations in 1984 January. Figures 3 and 4 represent the same data of Figures 1 and 2 but with this fake "earthquake" included.

IV. LIMITS ON THE GRAVITATIONAL WAVE BACKGROUND

In order to estimate the average excitation of the ${}_0S_2$ modes, the expected spectral line profiles were least-squares fitted to the data of Figure 2. These profiles were all taken to be $Q = 660$ Lorentzians, $S_L(\omega)$, convolved with the window function, $W(\omega)$, of the 30 day blocks of data, i.e.,

$$S(\omega) = \int_{-\infty}^{\infty} S_L(\omega')W(\omega - \omega')d\omega' , \tag{7}$$

where

$$S_L(\omega) = \frac{A}{1 + Q^2(\omega^2/\omega_0{}^2 - 1)^2} ,$$

is the Lorentzian profile corresponding to a mode with frequency ω_0 and given mechanical Q; and

$$W(\omega) = \frac{T}{2\pi}\left(\frac{\sin \omega T/2}{\omega T/2}\right)^2 ,$$

which corresponds to the rectangular windowing function of a data block of length T. There is no evidence of an average excitation of any of the five ${}_0S_2$ modes. If one assumes that the five modes are excited equally (such would be the case if the source were an isotropic background of GWs), the 1 σ upper limit on the total mean square acceleration at these frequencies is 1.6×10^{-24} m^2 s^{-4}. From equation 1, this corresponds to an upper limit to a GW background at 0.31 mHz of 1.7×10^{-5} J m^{-3} Hz.

The higher harmonic quadrupole mode which couples most strongly to GWs is the ${}_4S_2$ mode at a frequency of 1.72 mHz (Boughn and Kuhn 1984). Unfortunately this mode is nearly degenerate with the ${}_0S_{10}$ mode which couples strongly to earthquakes (see Fig. 1). Since the ${}_0S_l$ modes are primarily excited by earthquakes, the level of excitation of these modes is a good indication of earthquake activity. Figure 5 represents the weighted average of the 146 monthly spectra for which the total signal in the ${}_0S_l$ modes ($13 < l < 21$) was a minimum. Again there is no evidence of a background excitation of this mode. The 1 σ upper limit of the total mean square acceleration is 1.0×10^{-24} m^2 s^{-1}. By equation 1, the implied 1 σ

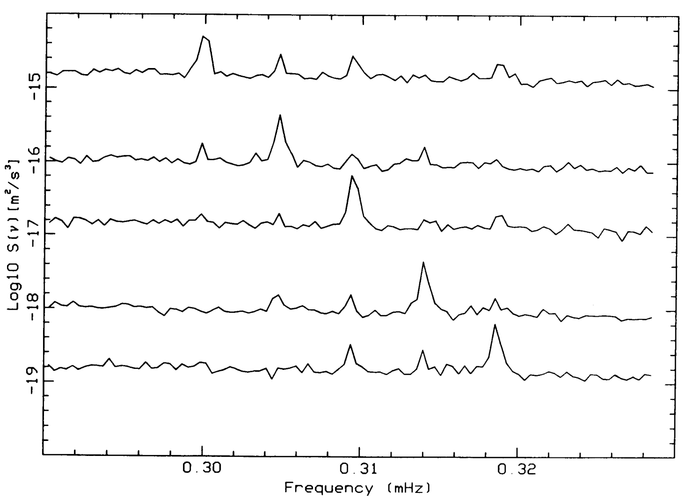

FIG. 4.—Same as Fig. 2 but with a fake signal included

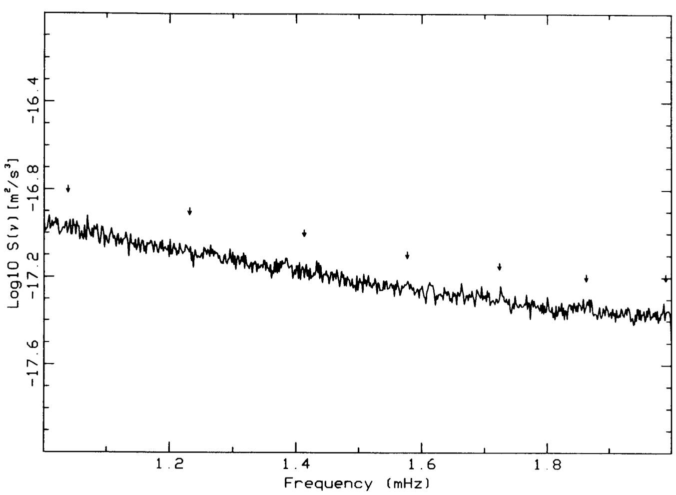

FIG. 5.—Average acceleration power spectrum for 146 earthquake quiet station months.

upper limit on a GW background at 1.72 mHz is 6.1×10^{-6} J m^{-3} Hz.

V. CONCLUSIONS

The analysis of 100 station years of quiet seismic data shows no evidence of the excitation of Earth by a stochastic background of gravitational radiation. The absence of excitation of the two quadrupole modes which couple most strongly to GWs (${}_0S_2$ and ${}_4S_2$) implies 1 σ upper limits on a GW background of 1.7×10^{-5} J m^{-3} Hz and 6.1×10^{-6} J m^{-3} Hz at frequencies of 0.31 mHz and 1.72 mHz, respectively. Although these limits are factors of 1300 and 33 lower than previous limits at these frequencies (Boughn and Kuhn 1984) they are both above the critical density of the universe per octave and are, therefore, relatively uninteresting from a cosmological point of view.

Since the background seismic noise that determines these limits is due to local barometric pressure fluctuations (Murphy and Savino 1975), it is conceivable that lower limits could be obtained by properly taking these fluctuations into account. The question is then, "What is the ultimate sensitivity of Earth as a gravitational wave detector?" The answer depends on the average background excitation of the above two modes by terrestrial sources. It seems unlikely that the limits at 1.72 mHz will improve significantly since the ${}_4S_2$ mode is degenerate with the earthquake sensitive ${}_0S_{10}$ mode. The ${}_0S_2$ mode on the other hand shows no sign of excitation although the expected average signal due to earthquakes is of the order of the observed limit. This situation improves if one edits out earthquake active months. The expected mean square acceleration for the quietest six months of 1984 is nearly a factor of 30 below the observed limit. Therefore, it seems as if there is still hope of using Earth as a millihertz GW detector if the local seismic noise can be corrected for; however, a remaining question to be answered is to what extent other terrestrial sources, e.g., barometric pressure fluctuations and ocean swell, excite the ${}_0S_2$ mode. Estimates of these effects are in progress.

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