reference frame

Microscopic irreversibility and chaos

Jerry Gollub and David Pine

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The origin of irreversible behavior is a persistent theme in physics. Since the fundamental microscopic laws, including both Newton's laws and quantum mechanics, are reversible (except for the weak interactions), the fact that most macroscopic systems behave irreversibly has long been recognized as an important issue.

The problem of understanding macroscopic irreversibility was solved more than 100 years ago by Ludwig Boltzmann, who recognized that systems evolve toward more probable states, namely, those that have a larger number of microscopic configurations for a given macroscopic state. This is such a powerful tendency that on the macroscopic level, fluctuations that go against it are so improbable as to be negligible. The result is a probabilistic explanation of macroscopic irreversibility and the second law of thermodynamics. (See the article by Joel Lebowitz in PHYSICS TODAY, September 1993, page 32.)

Nevertheless, Boltzmann did not explain the microscopic chaotic dynamics that leads to macroscopic irreversibility. Consider an imaginary gas of hard spheres that elastically collide with each other and obey the laws of Newtonian mechanics. It is perfectly conceivable that many microscopic states will never be visited, even in the age of the universe, and that some improbable states will persist, so the exploration required to achieve macroscopic irreversibility is not guaranteed. For example, a hard-sphere gas in a box in which all particles move parallel to the x direction so that they do not collide could persist forever. But, it is known that the hard-sphere gas is chaotic. Thus, on the average, a small perturbation to an initial configuration of particles becomes amplified exponentially over time. Chaos ensures that evolution to a representative sample of microstates occurs,[1] and that reversal of the velocities of all the particles does not in practice lead to a time-reversed motion. Therefore, both the microscopic and macroscopic behaviors of statistical systems are irreversible.

The strength of this sensitivity to initial conditions due to chaotic dynamics may be characterized by what are known as the Lyapunov exponents. They give the rates of exponential growth of the vector difference between two nearby trajectories along different directions in phase space.

In some fields of physics outside the domain of equilibrium statistical mechanics, similar considerations apply. For example, consider turbulence, which can produce irreversible mixing of a localized impurity in a fluid. The evolution of the fluid is governed by the Navier–Stokes equations, a set of deterministic nonlinear differential equations for the velocity field. Turbulence is generally understood to involve chaos arising from the nonlinear dynamics of these equations. (See the article by Gregory Falkovich and Katepalli Sreenivasan in PHYSICS TODAY, April 2006, page 43.) Therefore, a statistical description is necessary.

Flow at low Reynolds number

Direct experimental tests of reversible behavior are rare in physics. One can imagine looking at the evolution of a hard-sphere gas, but there is no way to reverse the velocities at an instant to investigate the time-reversed evolution. However, in fluid dynamics there is a way to do this, because the equations of fluid dynamics at low Reynolds number *Re* (that is, for sufficiently slow flow or high viscosity η) are in fact strictly reversible. These Stokes equations are obtained from the Navier–Stokes equations for incompressible fluids simply by omitting the nonlinear term and the time derivative of the velocity field:

$$-\nabla p + \eta \nabla^2 \mathbf{u} = 0$$
$$\nabla \cdot \mathbf{u} = 0.$$

The velocity field $\mathbf{u}(\mathbf{r}, \mathbf{t})$ is then governed simply by the pressure p and the boundary conditions on $\mathbf{u}(\mathbf{r}, \mathbf{t})$. In a simple shear flow between parallel plates, these equations imply that reversal of the boundary motion will instantly reverse the velocity at each position in the fluid.

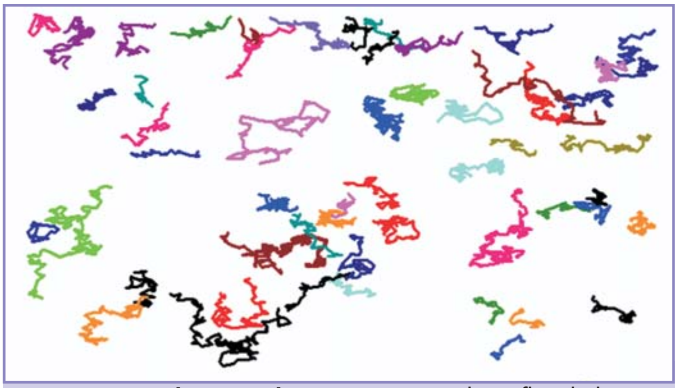
It may seem odd that the flow of a dissipative fluid can be reversible, since physicists usually associate reversibility with the conservative limit of classical mechanics. Yet in this driven system, reversibility occurs in the highly dissipative limit.

Half a century ago, G. I. Taylor made a movie that beautifully demonstrated the reversibility of low *Re* flow by inserting a small blob of dye into a very viscous fluid contained in the gap between two concentric cylinders, then rotating the inner cylinder many turns, thus stretching the dye into a thin filament about 50 cm long. Finally, the rotation was reversed, and the dye was miraculously reconstituted as a spherical blob, with only a slight amount of blurring due to Brownian motion at the edges of the blob. (See the online video[2] of Taylor's famous demonstration.)

A viscous suspension

Last year, we asked ourselves whether the same reversible behavior would occur if the fluid is a suspension of macroscopic particles rather than a pure viscous Newtonian fluid. A Newtonian fluid is one for which the shear force and strain rate or velocity gradient are proportional. Many common fluids, such as blood and paint, are actually suspensions. If we assume Brownian motion is negligible, the fluid should still follow the Stokes equations, though the equations would be complicated to solve exactly, since the correct no-slip boundary conditions would have to be satisfied on the surfaces of all the particles as well as on any exterior surfaces, such as the walls of the cylinders containing the fluid. One can reverse the motion of the fluid and the particles simply by reversing the motion of the inner cylinder, just as Taylor did in the demonstration just mentioned.

When we tried the experiment, we found that the motion of a dense viscous suspension is almost perfectly reversible, but only if the rotation prior to reversal is not too large. The appropriate dimensionless measure of rotation is the strain amplitude, the maximum azimuthal translation divided by the width of the gap between the cylinders. But if the strain amplitude exceeds a critical value, the motion actually becomes visibly irreversible, and particles fail to return to their starting positions.[3] In fact, they execute a random walk if the rotation is repeated and the particles are observed stroboscopically, that is, once per cycle of the rotation and reversal. The figure shows some of these apparently random trajectories. It looks like Brownian motion, but it is not due to thermal excitation, which is negligible for particles that are 200 microns in diameter.



Demonstration that particles in a reversing shear flow behave irreversibly. The trajectories shown are of tracked particles observed once per cycle. The maximum displacement is about twice the thickness of the fluid layer. Different colors indicate individual particles.

Why do the reversible equations of motion produce such irreversible behavior? To answer this question, our colleagues John Brady and Alex Leshansky performed accurate numerical simulations of the process.[3] They demonstrated that the motion is sensitive to initial conditions, and that the largest Lyapunov exponent for this process grows rapidly just where the experiments show that the motion becomes irreversible.

Thus, the irreversible particle trajectories in a viscous suspension flow resemble particle trajectories in a hard-sphere gas or tracers in turbulence. All three involve sensitivity to initial conditions or to perturbations anywhere along the path in phase space. The important difference is that the suspension case allows a clear experimental demonstration, because reversal of the boundary motion reverses the motion of all of the particles and the intervening fluid.

One remaining mystery is that we don't really know why there appears to be a threshold in the amount of rotation required to produce irreversibility. Other chaotic systems do not show a sharp change of this kind.

Note that the irreversibility of the trajectories of individual particles is microscopic, and therefore would lead inevitably to macroscopic irreversibility. For example, were we to repeat Taylor's experiment on a suspension by inserting a blob of dye, the blob should diffuse anomalously fast.

Other examples of irreversibility

Irreversibility of fluid motion also occurs at a high Reynolds number, but it is not surprising there because the equations are nonlinear and time dependent. For example, vorticity spreads irreversibly in a turbulent flow, in much the same way that tracer molecules spread in a solution.

Irreversibility and mixing can also occur at an arbitrarily low Reynolds number in a pure fluid, as Julio Ottino's elegant experiments have shown.[4] One example is the time-periodic flow in the annulus between counterrotating cylinders with the inner one off center. The equations for the stream function $\Psi(x, y)$, whose derivatives give the velocity components along x and y for fluid motion in two dimensions, turn out to be of the same form as Hamilton's equations of classical mechanics, as was pointed out years ago by Hassan Aref. The result is something equivalent to the chaos that occurs in classical mechanics (for example, in simple anharmonic oscillators), but in real space rather than in phase space. Chaotic mixing in these two-dimensional flows, where there are no added particles and the Reynolds number is small, is simply a consequence of the motion of fluid elements in a flow determined by the prescribed boundary motion. This pure fluid flow should also be irreversible.

These examples show that the irreversibility of microscopic trajectories of particles or fluid elements does not require that the dynamical equations violate time-reversal symmetry (though that can also happen, as in finite Reynolds number flows), or that the system be coupled to a source of external noise. Furthermore, fluid dynamics provides a natural context to explore reversibility experimentally.

Farther afield

Another well-known example of irreversibility mediated by chaos is the behavior of the solar system over long times.[5] The solar system has sensitivity to initial conditions, and it would presumably be irreversible as a result. However, the extent of irreversibility, and the possible existence of a threshold, would obviously have to be studied numerically.

Are there other areas of physics for which the origins of microscopic irreversibility can be usefully explored? These days, there is much interest in coherent quantum systems that could potentially be used for computing. One of the key difficulties in achieving useful computation is decoherence, in which interactions between a quantum system and its surroundings produce irreversible change. That occurs even though the laws of quantum mechanics governing the system plus its environment are reversible, as is the case for the suspension flow, where the fluid equations are reversible but the particles behave irreversibly.

The irreversibility due to chaos discussed in this column is quite different from the violation of time-reversal symmetry that occurs, for example, in the weak interactions governing the K-meson system. The microscopic irreversibility of particle trajectories in the viscous suspension arises from sensitivity to initial conditions. It does not require any fundamental asymmetry between future and past and provides a mechanism for both macroscopic and microscopic irreversibility. The experimental study of irreversibility can lead to unexpected results.

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References

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