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Magnetic fields from heterotic cosmic strings

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Large-scale magnetic fields are observed today to be coherent on galactic scales. While there exists an explanation for their amplification and their specific configuration in spiral galaxies—the dynamo mechanism—a satisfying explanation for the original seed fields required is still lacking. Cosmic strings are compelling candidates because of their scaling properties, which would guarantee the coherence on cosmological scales of any resultant magnetic fields at the time of galaxy formation. We present a mechanism for the production of primordial seed magnetic fields from heterotic cosmic strings arising from M theory. More specifically, we make use of heterotic cosmic strings stemming from $M5$-branes wrapped around four of the compact internal dimensions. These objects are stable on cosmological time scales and carry charged zero modes. Therefore a scaling solution of such defects will generate seed magnetic fields which are coherent on galactic scales today.

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I. INTRODUCTION

In this article we construct stable heterotic cosmic strings arising from suitably wrapped $M5$-branes, following [1]. We argue that a network of these strings could be responsible for the generation of primordial magnetic fields, as in the pion string case [2]. This gives a possible string theoretical explanation for the large-scale magnetic fields observed in the universe today.

In Sec. II we give the astrophysical motivation for the problem and explain why cosmic strings might be relevant to its resolution. In Sec. III we present as candidates the heterotic cosmic strings of [1]. In order for these strings to generate galactic magnetic fields, they must both be stable and support charged zero modes. We show that this is the case: in Sec. IV we find that in order for these strings to support charged zero modes a more general picture is required, in which the moduli of a large moduli space of M-theory compactifications are time dependent and evolve cosmologically. Stability and production of our cosmic string candidates is discussed in Sec. V, and the amplitude of the resulting fields given in Sec. VI. We end with a discussion.

II. PRIMORDIAL MAGNETIC FIELDS AND COSMIC STRINGS

The gaseous disk of the galaxy is known to contain a toroidal magnetic field with a strength of 3×10^{-6} G which is coherent on scales of up to a megaparsec [3–6]. These fields are believed to be ubiquitous in galaxies and galactic clusters. They have no contemporary source, and cannot be primordial since their decay time is 2 orders of magnitude less than the galactic lifetime of 10^{10} years [7]. In order for fields still to be present at late times, there must be some process that generates galactic flux continually. The likeliest suspect is the galactic dynamo.

Turbulent motions in the interstellar medium are rendered cyclonic by the nonuniform rotation of the gaseous disk of the galaxy. The so-called $\alpha\omega$ dynamo that results has been shown to be responsible for regeneration and amplification of the magnetic field of the galaxy [7–10].[1]

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[1] The classic texts on magnetohydrodynamics and dynamo theory are [4,11]. See also [12,13]. Widrow's review [6] is especially lucid and contains the key references.

However, the dynamo still requires seed primordial fields to amplify—there is no source term in the relevant hydromagnetic equation. The minimum required amplitude of these fields at the time of galaxy formation can be found to be 10^{-20} G [6].[2] Furthermore, they should be coherent on cosmological scales at the time of galaxy formation. For a fundamental process to be responsible for these seed fields, this coherence is a nontrivial condition. Galaxy formation occurs at very late times ($\sim t_{eq}$) from a particle physics perspective. Typical particle physics processes will create magnetic fields whose coherence length is limited by the Hubble radius at the time t_{pp} when the processes take place. A particle physics source that will scale appropriately so as to avoid this problem is given by cosmic strings.

These are topological defects formed during phase transitions as the universe cooled (in the case that the vacuum manifold $\mathcal{M}$ has a nontrivial first homotopy group) [15–17]. A network of these macroscopic strings will generically form, parametrized by a characteristic length scale $\xi(t)$ which expands with the expansion of the universe. Both infinitely long strings and loops will form. Sufficiently small loops can decay away via gravitational radiation, but the rate at which strings can chop each other off into loops is limited by the speed of light. What results is a scaling solution in which the string properties such as $\xi(t)$ are all proportional to the time passed. This has been confirmed by simulations [18–20] and implies that if cosmic strings can produce magnetic fields they will be coherent over galactic scales at the time of galaxy formation, as required.

Production of primordial magnetic fields from cosmic strings was proposed in [2], for the case of pion strings. These are global vortex line solutions of the effective QCD Lagrangian below the chiral symmetry breaking scale $T_c \sim 100$ MeV [21]. These pion strings couple to electromagnetism via anomalous Wess-Zumino–type interactions. Using the results of [22] for such a coupling, it was shown that pion strings could generate coherent seed magnetic fields greater than 10^{-20} G, provided the strings reach scaling soon enough.[3] The argument in [2] requires the existence of current on the pion strings. Such current will automatically be generated at the time of the phase transition provided that the strings admit charged zero modes, i.e. are superconducting [28].

Note that magnetic fields generated by cosmic strings can be inherited by galaxies both in models where they are seeded by cosmic string loops [29–31] and in models (supported by more recent simulations [32,33]) where most of the structure formation triggered by strings occurs in the wakelike overdensities behind long moving strings. The coherence length of these fields is then comparable to or larger than the regions which collapse to form galaxies. Provided that pion strings decay later than the time corresponding to a temperature of 1 MeV, this final correlation length will be of the size of a galaxy. Note that in this model, there is an upper cutoff on the scale of coherent magnetic fields. Magnetic fields on supergalactic scales can arise only as a random superposition of galactic scale fields, and hence the power spectrum of magnetic fields will be Poisson suppressed on these scales.

III. HETEROTIC COSMIC STRINGS

We begin by considering heterotic cosmic strings for phenomenological reasons and because charge is evenly distributed over them rather than being localized at the end points. However, fundamental heterotic strings were ruled out as candidates for cosmic strings by Witten in 1985 [34]. Although simple decay is ruled out because there are no open strings in the theory,[4] Witten argues that the fundamental heterotic string is actually an axionic string, and as a result is unstable.

Fundamental heterotic strings were also ruled out by Witten [34] as viable cosmic string candidates on tension grounds. In perturbative string theory about a flat background, the string tension is too large to be compatible with the existing limits [36].

A. Loopholes via *M*-theory and the Becker, Becker, and Krause construction

The possibility of obtaining stable cosmic superstrings was resurrected by Copeland, Myers, and Polchinski [37] (see also [38] and the review in [39]). The existence of extended objects of higher dimension, namely branes of various types, provides a way to overcome the instability problems pointed out by Witten [34], as we shall see for the heterotic string in particular. On the other hand, string tensions can in general be lowered by placing the strings

[2]This takes into account the amplification undergone by these fields during the collapse of gas clouds to form galaxies. It should be noted that this minimum could increase. Observations of microgauss fields in galaxies at a redshift of 2 shorten the time available for dynamo action and lead to a seed field as large as 10^{-10} G [6]. Similarly, imperfect escape of field lines may allow only a limited amplification of the mean field [14].

[3]The interaction of cosmic strings with magnetic fields has been discussed in many papers, starting with [23], but their possible connection to primordial galactic fields was first suggested in [24] and then elaborated on in [25]. The importance of the coherence length was not commented on until [2]. Note that a different mechanism of magnetogenesis from cosmic strings was proposed in [26], in which it was argued that vortices formed by cosmic string loops could produce magnetic fields by the Harrison-Rees effect. See [27] for a discussion of the difficulties of using the Harrison mechanism to create magnetic fields from topological defects. The approach here is rather to show that the strings produce the seed magnetic fields directly.

[4]Note that this is not necessarily the case for the $SO(32)$ heterotic string which can end on monopoles. This was pointed out by Polchinski [35].

in warped throats of the internal manifold and using the gravitational redshift to reduce the string tensions, so that this constraint no longer rules out all cosmic superstrings.

Using the axionic instability loophole presented in [37], Becker, Becker, and Krause [1] studied the possibility of cosmic strings in heterotic theory, pointing out that suitable string candidates can arise from wrapped branes in M theory. When compactified on a line segment $S^1/\mathbb{Z}_2$, M theory reduces to heterotic string theory [40]. Compactifying a suitable configuration to $3+1$ dimensions could give us heterotic cosmic strings in our world. Note that, because brane tensions are significantly lower than the fundamental string tension, the cosmic strings arising from such wrapped branes can also avoid the tension bound mentioned above.

There are two kinds of M-theory branes to consider as potential cosmic string candidates: $M2$- and $M5$-branes, wrapping 1- or 4-cycles, respectively, in the internal dimensions

Heterotic string theory is obtained by compactifying M theory on $S^1/\mathbb{Z}_2$, so the internal dimensions are naturally separated into x^{11} along the circle, and $x^4, \ldots, x^9 \in CY_3$ on the ten-dimensional boundaries of the space, which we can think of as $M9$-branes. Thus there are four possible wrapped-brane configurations, which can be labeled (following the notation of [1]) as $M2_\perp$, $M2_\parallel$, $M5_\perp$, and $M5_\parallel$, where the designations perpendicular and parallel refer to the brane wrapping and not wrapping the orbifold direction x^{11}, respectively. Their viability as cosmic string candidates is discussed below.

B. Wrapped $M2$-branes

There is no 1-cycle available in a Calabi-Yau threefold, so the $M2$-brane candidates can only wrap x^{11}. We can check their viability by comparing the tension of the resulting cosmic strings with the constraint given by anisotropy measurements of the cosmic microwave background (CMB):[5]

$$\mu G_N \leq 2 \times 10^{-7}, \tag{3.1}$$

where G_N is Newton's gravitational constant.

The $M2$-brane action is given by

$$S_{M2} = \tau_{M2} \int dt \int dx \int_0^L dx^{11} \sqrt{-\det h_{ab}} + \cdots, \tag{3.2}$$

where τ_{M2} is the tension of the brane, and h_{ab} denotes the world sheet metric. The 11-dimensional metric G_{IJ} of spacetime is found by considering the internal manifold to be compactified by the presence of G fluxes [43]. The result is

$$ds_{11}^2 = e^{-f(x^{11})} g_{\mu\nu} dx^\mu dx^\nu + e^{f(x^{11})}(g_{mn} dy^m dy^n + dx^{11} dx^{11}), \tag{3.3}$$

where

$$e^{f(x^{11})} = (1 - x^{11} Q_v)^{2/3}. \tag{3.4}$$

In the above $g_{\mu\nu}$ is the metric in our four-dimensional spacetime, and g_{mn} is the metric on the Calabi-Yau threefold. There is warping along the orbifold direction given by the function $f(x^{11})$, and Q_v is the two-brane charge. Making use of the above metric, we obtain from (3.2) the following cosmic string action:

$$\begin{aligned} S_{M2} &= \mu_{M2} \int dt \int dx \sqrt{-g_{tt} g_{xx}} + \cdots, \\ \mu_{M2} &= \tau_{M2} \int_0^L dx^{11} e^{-f(x^{11})/2} \\ &= \frac{3\tau_{M2}}{2Q_v}[1 - (1 - LQ_v)^{2/3}]. \end{aligned} \tag{3.5}$$

Upon evaluation, this gives a brane tension of

$$\mu_{M2} \approx 9(2^{10}\pi^2)^{1/3} M_{\rm GUT}^2, \tag{3.6}$$

which is too large to satisfy the bound (3.1). Thus wrapped $M2$-branes are ruled out as candidates for heterotic cosmic strings. However, they are stable (see [1]). If produced in a cosmological context, they would therefore have disastrous consequences.

C. Wrapped $M5$-branes: Tension

For the case of the $M5$-brane, there are two possible types of configurations. Following [1] we label them $M5_\parallel$ and $M5_\perp$. The $M5_\parallel$-brane is confined to the ten-dimensional boundary of the space, wrapping a 4-cycle Σ_4, while the $M5_\perp$-brane wraps x^{11} and a 3-cycle Σ_3. By similar analyses to those outlined above, one obtains the brane action for the parallel five-brane:

$$S_{M5_\parallel} = \tau_{M5} \int dt dx \int_{\Sigma_4} d^4y \sqrt{-\det h_{ab}} + \cdots, \tag{3.7}$$

where τ_{M5} is the brane tension. The effective string tension from the point of view of four-dimensional spacetime is given by

$$\mu_{M5_\parallel} = 64\left(\frac{\pi}{2}\right)^{1/3}\left(1 - \frac{x^{11}}{L_c}\right)^{2/3} M_{\rm GUT}^2 r_{\Sigma_4}^4, \tag{3.8}$$

where r_{Σ_4} measures the mean radius of the 4-cycle Σ_4 in units of the inverse GUT scale. L_c is a critical length of the $S^1/\mathbb{Z}_2$ interval determined by G_N.[6]

[5] This limit is given in [36,41] where WMAP and SDSS data was used. A tighter bound of 10^{-8} is suggested by analysis of limits on gravitational waves from pulsar timing observations [42]. However, these pulsar bounds are not robust since they depend sensitively on the distribution of cosmic string loops which is known rather poorly.

[6] See [43,44] for the derivations.

Similarly, for the orthogonal five-brane one obtains

$$S_{M5_\perp} = \tau_{M5} \int dt dx \int_0^L dx^{11} \int_{\Sigma_3} d^3y \sqrt{-\det h_{ab}} + \cdots, \tag{3.9}$$

and the associated cosmic superstring tension is

$$\mu_{M5_\perp} = \frac{1152}{5} \frac{\pi^{1/3}}{2} M_{\rm GUT}^2 r_{\Sigma_3}^3, \tag{3.10}$$

where r_{Σ_3} measures the mean radius of the 3-cycle Σ_3 in units of the inverse GUT scale. Although there is some dependence on the size of the wrapped space, it is not hard for the $M5_\parallel$-brane to pass the CMB constraint. With a little more difficulty, the $M5_\perp$ brane also passes this test [although the numerical coefficient given in (3.23) of [1] is about an order of magnitude too small].

D. Wrapped $M5$-branes: Stability

The next check is a stability analysis, which shows that only the $M5_\parallel$-brane is stable. The reason is that axionic branes are unstable [34]. The massless axion that is responsible for this instability can only be avoided in the case of the $M5$-brane on the boundary: $M5_\parallel$. The argument is presented in detail in [1] and is sketched below (see also [37,38]).

To begin with, the presence of a massless axion is generally implied by the existence of the branes. $M5$-branes are charged under C_6 (the Hodge dual to C_3 in 11 dimensions). This form descends to C_2 in the four-dimensional theory and, via

$$\star\, dC_2 = d\phi, \tag{3.11}$$

this implies the presence of an axionic field. However, the presence of the $M9$ boundaries leads to a modification of $G = dC_3$ on the boundaries. Together with appropriate $U(1)$ gauge fields, this leads to a coupling of C_2 to the gauge fields. This amounts to a Higgsing of the gauge field which then acquires a mass given by the axion term.

To see how this happens, recall that, because of the presence of the boundaries on which a ten-dimensional theory lives, an anomaly cancellation condition must be satisfied. Writing the ten-dimensional anomaly as $I_{12} = I_4 I_8$ we require for anomaly cancellation the existence of a two-form B_2 such that $H = dB_2$ satisfies

$$dH = I_4. \tag{3.12}$$

In addition, it is required that the interaction term

$$\Delta L = \int B_2 \wedge I_8 \tag{3.13}$$

be present [40]. In M theory the four-form I_4 is promoted to a five-form I_5, and although $dG = 0$ (a Bianchi identity) in the absence of boundaries, we must have

$$dG \sim \delta(x^{11}) dx^{11} I_4 \tag{3.14}$$

in the presence of boundaries. Thus, the Bianchi identity acquires a correction term which turns out to be [40]

$$dG = c\kappa^{2/3} \delta\left(\frac{x^{11}}{L}\right)\left(d\omega_Y - \frac{1}{2} d\omega_L\right), \tag{3.15}$$

written in terms of the Yang-Mills three-form ω_Y and the Lorentz Chern-Simons three-form ω_L given by

$$d\omega_Y = {\rm tr}\, F \wedge F; \qquad d\omega_L = {\rm tr}\, R \wedge R. \tag{3.16}$$

Then

$$G = dC_3 + \frac{c}{2} \kappa^{2/3}\left(\omega_Y - \frac{1}{2}\omega_L\right)\epsilon(x^{11}) \wedge dx^{11}$$

which implies

$$H = dB_2 - \frac{c}{2L} \kappa^{2/3}\left(\omega_Y - \frac{1}{2}\omega_L\right). \tag{3.17}$$

It follows that $H \wedge \star H$ contains the term

$$(\omega_Y - \tfrac{1}{2}\omega_L) \wedge dC_6 \tag{3.18}$$

which upon integration (and integrating by parts) yields

$$\int C_6 \wedge \left({\rm tr}\, F \wedge F - \frac{1}{2} {\rm tr}\, R \wedge R\right). \tag{3.19}$$

Note that C_6 is in the $M5$-brane directions here.

From earlier work we know the gauge group is generically broken to something containing a $U(1)$ factor, so there exists some F_2 on the boundary. Then the 11D action is

$$S_{\rm 11D} = -\frac{1}{2 \times 7! \kappa_{11}^2} \int_{\mathcal{M}^{11}} |dC_6|^2 + \frac{c}{2\kappa_{11}^{4/3}} \int_{\mathcal{M}^{10}} C_6 \wedge {\rm tr}\, F \wedge F - \frac{1}{4g_{10}^2} \int_{\mathcal{M}^{10}} |F|^2 \tag{3.20}$$

which dimensionally reduces to

$$S_{\rm 4D} = -\frac{1}{2} \int_{\mathcal{M}^4} |dC_2|^2 + m \int_{\mathcal{M}^4} C_2 \wedge F_2 - \frac{1}{2} \int_{\mathcal{M}^4} |F_2|^2, \tag{3.21}$$

where

$$m \propto \frac{L_{\rm top}^4}{V^{1/2} V_h^{1/2}}, \tag{3.22}$$

V being the CY volume averaged over the $\frac{S^1}{\mathbb{Z}_2}$ interval and V_h the CY volume at the boundary. $L_{\rm top}$ is a length parameter defined by

$$\int_{\mathcal{M}^{10}} C_6 \wedge {\rm tr}(F \wedge F_2) = L_{\rm top}^4 \int_{\mathcal{M}^4} C_2 \wedge F_2.$$

The equations of motion for A_1 and C_2 are found to be

$$d\star_4 dA_1 = -mdC_2; \tag{3.23}$$

$$d\star_4 dC_2 = -mF_2. \tag{3.24}$$

(3.24) is solved by taking $dC_2 = \star(d\phi - mA_1)$ which gives

$$d\star dA_1 = \star(-md\phi + m^2 A_1). \tag{3.25}$$

For the ground state in which $\phi = 0$ or by picking a gauge which sets $d\phi = 0$, this result shows that A_1 has acquired a mass m:

$$A_1 \to A_1 - \frac{d\phi}{m}. \tag{3.26}$$

The $U(1)$ gauge field has swallowed the axion ϕ and become massive. The theory no longer contains an axion.

In order for this anomaly cancellation mechanism (which swallows the axion and thus eliminates the instability of the strings) to work, the gauge field must be on the boundary and thus the brane must be parallel to the boundary. Thus, only the $M5_{\|}$-brane is stabilized, and the $M5_{\perp}$-brane remains unstable.

IV. CHARGED ZERO MODES ON THE STRINGS

We now need to argue for the existence of charged zero modes (we will focus on fermionic zero modes) on the strings arising from wrapped $M5_{\|}$-branes. In 1 + 1 dimensions, the degrees of freedom of free fermions and free bosons match, and the corresponding conformal field theories (CFTs) can be shown to be equivalent. This is not the case in higher dimensions, where spin degrees of freedom distinguish between them. This observation is at the heart of bosonization, the process of going from a fermionic basis to a bosonic basis. In evaluating the superconductors on the string resulting from the wrapped $M5$-brane, we find that the correct basis is a charged fermionic one, implying fermionic superconductivity.

Here we derive the coupling to electromagnetism that can arise on the world sheet of the heterotic cosmic string and argue using inverse bosonization (fermionization) that this can be recast in a more familiar form by writing it in terms of fermions. What results is an explicit kinetic term for charged fermions on the world sheet.

A. Coupling to electromagnetism

Consider a wrapped $M5_{\|}$-brane. It can be taken to be along the following directions:

$$M5_{\|} \quad 0 \quad 1 \quad 4 \quad 5 \quad 6 \quad 7.$$

Let the 0, 1 coordinates be labeled by x and the remaining coordinates wrapped on Σ_4 be labeled by y. The massless field content on the five-brane world volume is given by the tensor multiplet $(5\phi, B^+_{mn})$ [45–47], where the scalars correspond to excitations in the transverse directions and the tensor is antisymmetric and has anti-self-dual field strength $H_3 = dB^+$. Thus it has $3 = \frac{1}{2}\times {}^4C_2$ degrees of freedom which, together with the scalars, make up the required eight bosonic degrees of freedom.[7]

The field strength H_3 couples to C_3, the bulk three-form field sourced electrically by the $M2$-brane and magnetically by the $M5$-brane, as given in [48]:

$$S = -\frac{1}{2}\int d^6\sigma\sqrt{-h}\Big[h^{ij}\partial_i X^M \partial_j X^N g_{MN} + \frac{1}{2}h^{ij}h^{jm}h^{kn}(H_{ijk} - C_{ijk})(H_{lmn} - C_{lmn}) - 4\Big], \tag{4.1}$$

which can be rewritten in terms of differential forms as

$$S = -\frac{1}{2}\int d^6\sigma\sqrt{-h}(h^{ij}g_{ij} - 4) - \frac{3}{2}\int (H_3 - C_3)\wedge\star(H_3 - C_3). \tag{4.2}$$

Here $i, j = 0, 1, \ldots, 5$ are indices on the brane world volume and $M, N = 0, \ldots, 9, 11$ are indices in the full eleven-dimensional theory. g_{ij} is the pullback of the eleven-dimensional metric, C_{ijk} is the pullback of the eleven-dimensional three-form, and h is the auxiliary world volume metric. Explicitly,

$$g_{ij} = \partial_i X^M \partial_j X^N g^{(11)}_{MN}; \tag{4.3}$$

$$C_{ijk} = \partial_i X^M \partial_j X^N \partial_k X^P C^{(11)}_{MNP}. \tag{4.4}$$

B^+ and C_3 are both functions of y as well as x. To find the massless modes on the string upon compactification on X, we decompose them in terms of harmonic forms. For a harmonic differential form β on a closed compact manifold (such as Σ_4), we have $d\beta = d\star\beta = 0$. The two-form is decomposed as

$$B^+ = \phi^a(x)\otimes\Omega^a_2(y) + b_2(x)\otimes\Phi(y); \tag{4.5}$$

$$dB^+ = d\phi^a(x)\otimes\Omega^a_2(y), \tag{4.6}$$

where a runs over the two-cycles on the Σ_4 which the $M5$-brane wraps.[8] We have taken $H^1(\Sigma_4) = 0$ for simplicity. Ω^a_2 are the harmonic two-forms on Σ_4, and b_2 is a two-

[7] A $D = 11$ Majorana spinor has 32 real components, which are reduced to 16 by the presence of the $M5$-brane. This means the $M5$-brane theory will have 16 fermionic zero modes and eight bosonic zero modes [46].

[8] We take Ω^a_2 to be anti-self-dual, so that $a = 1, \ldots, b_-$, where we have chosen a basis of $H^2(\Sigma_4)$ made of (b_+) forms which are entirely self-dual and (b_-) forms which are entirely anti-self-dual. This imposes the property of anti-self-duality mentioned earlier for the two-form living on the five-brane. (Clearly then, Dim $H^2(\Sigma_4) = b_- + b_+$.)

form in the 0,1 directions. Similarly we want C_3 to be decomposable as

$$C_3 = A^a(x) \otimes \tilde{\Omega}_2^a(y) + \varphi^p(x) \otimes \tilde{\Omega}_3^p(y), \tag{4.7}$$

where the $\tilde{\Omega}_2^a$ are now harmonic two-forms on the CY base, as this decomposition could give rise to the required $U(1)$ gauge fields A^a in x space. This time a runs over the $h^{(1,1)}$ possible two-cycles on the internal space, while p runs over the $2h^{(2,1)}$ possible three-cycles. We have also denoted the harmonic three-forms by $\tilde{\Omega}_3^q$.

B. Moduli space of M-theory compactifications

The M-theory description of the $E_8 \times E_8$ string that we have been using so far now leads to the following puzzle. To allow a decomposition of the three-form field of the kind that we want means that the background C_3 flux would have to be switched on parallel to the $M5_{\parallel}$-brane. This is impossible for M-theory compactified on $S^1/\mathbb{Z}_2$ because the $\mathbb{Z}_2$ projection demands

$$C_3 \rightarrow -C_3, \tag{4.8}$$

and therefore all components of the background G flux with no legs along the $S^1/\mathbb{Z}_2$ direction are projected out! Our naive compactification of M-theory on $CY \times S^1/\mathbb{Z}_2$ therefore cannot give rise to charged modes propagating on the string, making the situation at hand rather subtle.

However, in a cosmological setting an $E_8 \times E_8$ heterotic string in the limit of strong coupling cannot simply be described by a time-independent M-theory background. Instead the description should be in terms of a much bigger moduli space of M-theory compactifications, with the moduli themselves evolving with time. Specifically, we require a large moduli space of M-theory compactifications that would include the heterotic compactification above, at least for $t = 0$. Such a picture can be motivated from the well-known F-theory/heterotic duality which relates F-theory compactified on a K3 manifold to heterotic string theory compactified on a two-torus T^2 [49–51]. From here it follows immediately that M-theory compactified on K3 will be dual to heterotic string theory compactified on a three-torus T^3. Fibering both sides of the duality by another T^3 gives us

$$\begin{aligned} &M \text{ theory on a } G_2 \text{ holonomy manifold} \\ &\quad \equiv \text{heterotic string theory on } \mathcal{M}_6, \end{aligned} \tag{4.9}$$

where the G_2 holonomy manifold is a seven-dimensional manifold given by a nontrivial T^3 fibration over a K3 base, and $\mathcal{M}_6$ is a six-dimensional manifold given by a nontrivial T^3 fibration over a T^3 base. Note that $\mathcal{M}_6$ is not in general a CY space. This duality has been discussed in the literature [52].

To confirm that there exists a point in the M-theory moduli space that describes the $E_8 \times E_8$ heterotic string, one needs to study the degeneration limits of the elliptically fibered base K3 (which can be written as a T^2 fibration over a P^1 base). Elliptically fibered K3 surfaces can be described by the family of elliptic curves (called Weierstrass equations)

$$y^2 = x^3 + f(z)x + g(z), \tag{4.10}$$

where (x, y) are the coordinates of the T^2 fiber of K3 and z is a coordinate on P^1, and f and g are polynomials of degree 8 and 12, respectively. Different moduli branches exist for which the modulus τ of the elliptic fiber is constant [53]. Gauge symmetries arise from the singularity types of the fibration on these branches. $E_8 \times E_8$ *can* be realized: The specific degeneration limit of K3 that produces an $E_8 \times E_8$ heterotic string corresponds to the Weierstrass equation [50,53]:

$$y^2 = x^3 + (z - z_1)^5(z - z_2)^5(z - z_3)(z - z_4). \tag{4.11}$$

The two zeroes of order 5 each give rise to an E_8 factor, while the simple zeroes give no singularity.[9]

Given the existence of such a point in the moduli space of M-theory compactification, the future evolution of the system will in general take us to a different point in the moduli space. The picture that emerges from here is rather interesting. We start with heterotic $E_8 \times E_8$ theory. The strong coupling effects take us to the M-theory picture. From here cosmological evolution will drive us to a general point in the moduli space of G_2 manifolds. In fact, no matter where we start off, we will eventually be driven to some point in the vast moduli space of G_2 manifolds.

With M-theory compactified on a G_2 manifold, turning on fluxes becomes easy. However, there are still a few subtleties that we need to address. First, in the presence of fluxes we only expect the manifolds to have a G_2 *structure* and not necessarily G_2 holonomy.[10] Thus, the moduli space becomes the moduli space of G_2 structure manifolds.[11] Second, due to Gauss' law constraint we will have to consider a noncompact seven manifold, much like

[9]This point in the moduli space of the M-theory compactification could as well be locally an $S^1/\mathbb{Z}_2$ fibration over a six-dimensional base $\tilde{\mathcal{M}}_6$ (we have not verified this here). Then the theory is dual to the $E_8 \times E_8$ heterotic string compactified on $\tilde{\mathcal{M}}_6$, and there is a clear distinction between $M5_{\parallel}$ and $M5_{\perp}$. Our earlier stability analysis could then be used to eliminate $M5_{\perp}$.

[10]For details on G_2 structure, see for example [54].

[11]As should be clear, we are no longer restricted to K3 fibered cases only. This situation is a bit like that of conifold transitions where we go from one CY moduli space to another in a cosmological setting governed by rolling moduli [55]. Furthermore, the constraint of G_2 structure comes from demanding low-energy supersymmetry. Otherwise we could consider any seven manifold.

the one considered in [56].[12] Finally, since our $M5$-brane wraps a four-cycle inside the seven-manifold and we are switching on G fluxes parallel to the directions of the wrapped $M5$-brane, we need to address the concern of [57] that this is not permitted.

In the presence of a G flux on the four-cycle a wrapped $M5$-brane has the following equation of motion:[13]

$$dH_3 = G. \tag{4.12}$$

For a four-cycle with no boundary this implies $G = 0$, as in [57]. However, our case is slightly different. We have a wrapped $M5$-brane on a four-cycle, but the G flux has two legs along the wrapped cycle (the $x^{4,5}$ directions, say) and two legs in the $x^{0,1}$ directions. Therefore the G flux is defined on a *noncompact* four-cycle and we can turn it on if we modify the above equation (4.12) by inserting n $M2$-branes ending on the wrapped $M5$-brane. The $M2$-branes end on the $M5$ in small loops of string in the $x^{4,5}$ directions, with their other ends at some point along the noncompact direction inside the seven-manifold, which the $M2$-branes are extended along. These strings will change (4.12) to

$$dH_3 = G - n\sum_{i=1}^{n} \delta^4_{\mathbf{W}^i}, \tag{4.13}$$

where the $\delta^4_{\mathbf{W}^i}$ denote the localized actions of n world sheets on the $M5$-brane.[14] Then G need no longer be vanishing. In fact,

$$\int_{\tilde{\Sigma}_4} G = n, \tag{4.14}$$

where $\tilde{\Sigma}_4$ is the noncompact 4-cycle. This way we see that (a) we can avoid the $\mathbb{Z}_2$ projection (4.8) by going to a generic point in the moduli space of G_2-structure manifolds, and (b) we can switch on a nontrivial G flux along an $M5$-brane wrapped on a noncompact 4-cycle. Using the decompositions (4.5) and (4.7) we can now factorize the interaction term:

$$\begin{aligned} S_{\rm int} &= -\frac{3}{2}\int (H_3 - C_3)\wedge\star(H_3 - C_3) + \cdots \\ &= -\frac{3}{2}\int (dB^+ - C_3)\wedge\star(dB^+ - C_3) + \cdots \\ &= -\frac{3}{2}\int (d\phi^a - A^a)\wedge\star(d\phi^b - A^b)\otimes\Omega_2^a\wedge\star\tilde{\Omega}_2^b \\ &\quad -\frac{3}{2}\int d^2x\sqrt{-h_x}\varphi^p\varphi^q\Omega_3^p\wedge\star\tilde{\Omega}_3^q + \cdots \end{aligned} \tag{4.15}$$

where the dotted terms above involve the n tadpoles coming from the world volume strings. These tadpoles would be proportional to ϕ^a. The variables h_x and h_y denote the determinants of the world volume metrics along the x and y directions, respectively. We are interested in the coupling to electromagnetism, so we focus on the first term of (4.15) and take the number of 2-cycles on Σ_4 to be 1.[15] Then we have

$$S_{\rm int} = -\frac{3}{2}\kappa\int d^2\sigma|d\phi - A|^2\sqrt{-h_x} + \cdots, \tag{4.16}$$

where

$$\kappa = \int_y \Omega_2\wedge\star\Omega_2 \tag{4.17}$$

is a constant factor.[16]

C. Fermionization

The coupling in (4.16) implies that the action can be expressed more conveniently as one generating fermionic superconductivity along the string. We can see this by rewriting the term in terms of fermions, using a process known as fermionization.

Fermionization[17] is possible because of the equivalence in $1 + 1$ dimensions of the CFTs of $2n$ Majorana fermions and n bosons.[18]

[12]Note that although the seven manifold is noncompact, the six-dimensional base is always compact here. Thus, our earlier arguments depending on the existence of closed compact cycles on a CY_3 still hold, for an undetermined number of such cycles on some compact six-dimensional base. This is a construction we are free to choose.

[13]This can be seen from (4.2): one has to find the equation of motion for B^+ and then impose anti-self-duality of H_3.

[14]From the type IIB point of view, this is analogous to the baryon vertex with spikes coming out from the wrapped $D3$-brane on a $\mathbf{S}^3$ with H_{RR} fluxes in the geometric transition setup [58].

[15]In the presence of multiple 2-cycles we will have more Abelian fields. This does not change the physics of our discussion here.

[16]Note that there would also be non-Abelian gauge fields coming from G fluxes *localized* at the singularities of the G_2-structure manifolds in the limit where some of the singularities are merging. The G flux that we have switched on is nonlocalized. This picture is somewhat similar to the story developed in [59] where heterotic gauge fields were generated from localized G fluxes on an eight manifold. In a time-dependent background all these fluxes would also evolve with time, but for our present case it will suffice to assume a slow evolution so that the gauge fields (Abelian and non-Abelian) do not fluctuate very fast.

[17]Canonical references are [60–62]. Reference [14] of [63] gives a comprehensive list of the early references. A useful textbook treatment is given in [64].

[18]This has been shown to hold in the infinite volume limit as well as in the finite volume case, where care must be taken to match the boundary conditions correctly [65]. Our long cosmic strings correspond to the infinite volume case.

The correlator for the bosonic field can be found from the action,[19]

$$S_B = \frac{1}{4\pi}\int d^2z\partial X^\mu(z,\bar{z})\bar{\partial}X^\nu(z,\bar{z}), \tag{4.18}$$

to be

$$\langle X^\mu(z)X^\nu(w)\rangle = -\eta^{\mu\nu}\ln(z-w); \tag{4.19}$$

$$\langle X^\mu(z)\partial X^\nu(w)\rangle = \eta^{\mu\nu}\frac{1}{(z-w)}; \tag{4.20}$$

$$\langle \partial X^\mu(z)\partial X^\nu(w)\rangle = -\eta^{\mu\nu}\frac{1}{(z-w)^2}, \tag{4.21}$$

where z and w are local complex coordinates on the world sheet and the correlators are all for the holomorphic (left-moving) parts of the bosonic fields only. The kinetic term for Majorana fermions on the world sheet is

$$S_F = \frac{1}{4\pi}\int d^2z(\psi^\mu\bar{\partial}\psi_\mu + \tilde{\psi}^\mu\partial\tilde{\psi}_\mu). \tag{4.22}$$

The fields ψ and $\tilde{\psi}$ are holomorphic and antiholomorphic, respectively, with the holomorphic correlator given by

$$\langle \psi^\mu(z)\psi^\nu(w)\rangle = \eta^{\mu\nu}\frac{1}{(z-w)}. \tag{4.23}$$

Equivalently we could write the action and correlators in terms of

$$\psi = \frac{1}{\sqrt{2}}(\psi^1 + \imath\psi^2), \qquad \bar{\psi} = \frac{1}{\sqrt{2}}(\psi^1 - \imath\psi^2), \tag{4.24}$$

as

$$S_F = \frac{1}{4\pi}\int d^2z(\bar{\psi}\,\bar{\partial}\,\psi + \psi\bar{\partial}\,\bar{\psi}) \tag{4.25}$$

(writing the holomorphic terms only). Then

$$\langle \psi(z)\bar{\psi}(w)\rangle = \frac{1}{(z-w)}.$$

These correlators lead one to make the identification

$$\psi(z) \equiv e^{\imath\phi(z)}; \qquad \bar{\psi}(z) \equiv e^{-\imath\phi(z)}, \tag{4.26}$$

where ϕ is the holomorphic part of one bosonic field. Now we consider the operator product expansions (OPEs) [64],

$$\begin{aligned} e^{\imath\phi(z)}e^{-\imath\phi(-z)} &= \frac{1}{2z} + \imath\partial\phi(0) + 2zT_B^\phi(0) + \cdots; \\ \psi(z)\bar{\psi}(-z) &= \frac{1}{2z} + \psi\bar{\psi}(0) + 2zT_B^\psi(0) + \cdots, \end{aligned} \tag{4.27}$$

where T_B^ϕ and T_B^ψ are the energy-momentum tensors arising from the actions (4.18) and (4.22):

$$T_B = -\tfrac{1}{2}\partial X^\mu\partial X_\mu - \tfrac{1}{2}\psi^\mu\partial\psi_\mu. \tag{4.28}$$

The identification (4.26) implies that the OPEs (4.27) should be equivalent, since all local operators in the two theories can be built from operator products of the fields being identified. This implies that the energy-momentum tensors of the two theories must be the same, allowing us to identify the theories as CFTs. This allows us to rewrite the kinetic term for n scalars as the kinetic term of a theory containing $2n$ fermions. Furthermore, we have the identification

$$\psi\bar{\psi} \equiv \imath\partial\phi. \tag{4.29}$$

We can now rewrite our wrapped M-brane term

$$\begin{aligned} |d\phi - A|^2 &= (\partial_\mu\phi - A_\mu)(\partial^\mu\phi - A^\mu) \\ &= \partial_\mu\phi\partial^\mu\phi - A_\mu\partial^\mu\phi - A^\mu\partial_\mu\phi + A^2 \end{aligned}$$

in a fermionic basis:[20]

$$\begin{aligned} |d\phi - A|^2 &= 2(\bar{\psi}\,\bar{\partial}\,\psi + \psi\bar{\partial}\,\bar{\psi}) + 2\imath A\psi\bar{\psi} + 2A\bar{A} \\ &= 2\psi_1\left(\bar{\partial} + \frac{\imath}{2}A\right)\psi_1 + 2\psi_2\left(\bar{\partial} + \frac{\imath}{2}A\right)\psi_2 \\ &\quad + 2A\psi_1\psi_2 + 2A\bar{A}, \end{aligned} \tag{4.30}$$

which makes it clear that the world sheet supports charged fermionic modes. Here A and $\bar{A}$ are defined in terms of components as in (4.24). Each boson is replaced by one ψ fermion and one $\bar{\psi}$ fermion at the same point, moving left at the speed of light, and carrying charge as shown explicitly by (4.30). This proves the existence of charged fermionic zero modes on the string obtained by suitably wrapping an $M5$-brane. Note that [28] gives a similar discussion, relating a theory describing charged fermionic

[19]We use the conventions of Polchinski [64], working in units where $\alpha' = 2$.

[20]We make use of the fact that ϕ is holomorphic, as discussed below.

zero modes trapped on a string to a bosonized dual with an interaction of the form $|d\phi - A|^2$.

One might worry that the above analysis should hold equivalently for the antiholomorphic part of the bosonic fields, leading to an equal number of right-moving fermionic modes. This is not the case, since ϕ is in fact holomorphic. From the anti-self-duality of dB^+ it follows that $d\phi = - \star d\phi$ in 1 + 1 dimensions.[21] Writing $d\phi$ as $(\partial + \bar{\partial})\phi$, one can show that $\bar{\partial}\phi = 0$ is implied by the anti-self-duality condition. This is just the condition that ϕ does not depend on $\bar{z}$, i.e. it is holomorphic or, in world sheet terms, left moving.

V. STABILITY AND PRODUCTION

A. Axionic stability

Finally, we should argue that the axionic instability is also removed for our case. This can be easily seen either directly from M-theory or from its type IIA limit. From a type IIA point of view the wrapped $M5$-brane can appear as a $D4$-brane or an $NS5$-brane in ten dimensions depending on which direction we compactify in M-theory. First, assume that the four-cycle Σ_4 on which we have the wrapped $M5$-brane is locally of the form $\Sigma_3 \times S^1$. Then M-theory can be compactified along the S^1 direction to give a wrapped $D4$-brane on Σ_3 in ten dimensions.[22] We can now eliminate the axion following Becker, Becker, and Krause [1]. The axion here appears from the $D4$-brane source; i.e., the five-form Ramond-Ramond charge C_5. This form descends to an axion in four dimensions exactly as we discussed before (dC_5 descends to dC_2 in four dimensions, which in turn is Hodge dual to $d\phi$, the axion). What are the gauge fields that will eat the axion? In the case of [1] the gauge fields arose on the ten-dimensional boundary. Here instead of the boundary, we can insert coincident $D8$ branes[23] that allow gauge fields to propagate on their world volume Σ_8. Therefore the relevant parts of the action are

$$- \frac{1}{\kappa_{10}^2} \int |dC_5|^2 + \mu_8 \int_{\Sigma_8} C_5 \wedge \mathrm{tr}\, F \wedge F - \frac{1}{g_{\mathrm{YM}}^2} \int_{\Sigma_8} |F|^2 \tag{5.1}$$

which dimensionally reduce to an action similar to (3.21). This implies that the $D8$-brane gauge fields can eat up the axion to become heavy, and in turn eliminate the axionic instability. One subtlety with this process is the global $D8$-brane charge cancellation once we compactify. In fact, a similar charge cancellation condition should also arise for the $M2$-branes that we introduced earlier to allow nontrivial fluxes on the $M5$ branes. We need to keep one of the internal directions noncompact to satisfy Gauss' law.[24]

If instead we dimensionally reduce in a direction orthogonal to the wrapped $M5$ brane, then one can show that it is impossible to eliminate the axionic instability by the above process. There might exist an alternative way to eliminate the axionic instability, but we have not explored it here.

B. Stability and superconductivity

At this point we pause to discuss the different types of cosmic strings permitted and the question of whether or not they can be superconducting. In general, cosmic strings can be either global (as in the case of Brandenberger and Zhang [2]) or local (as in the case of Becker, Becker, and Krause [1]) [35]. Superconductivity can also arise in two ways [28,67]. Global strings can be superconducting thanks to an anomalous term of the form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu a)^2 - F \wedge \star F - \frac{e^2}{32\pi^2}\left(\frac{a}{f}\right) F \wedge F, \tag{5.2}$$

which causes charge to flow into the string, as explored by Kaplan and Manohar [22] (earlier references are [68,69]). For local strings (which are local with respect to the axion), this term is no longer gauge invariant. Superconductivity is still possible if charged zero modes, either fermionic or bosonic, are supported on the gauge strings [28]. In that case a coupling of the form of (4.16) or (4.30) exists on the world sheet. As we have seen, although the heterotic cosmic strings constructed by Becker, Becker, and Krause [1] are local, they are not superconducting. A more general setup is required in order for fermionic zero modes to be permitted, which is what we have constructed. Thus, ours are local superconducting strings, where the superconductivity is clearest in a fermionic basis, as in (4.30).

C. Production of $M5_{\parallel}$-branes

Whether strings or branes of a particular type will be present at late cosmological times relevant to the generation of seed galactic magnetic fields will depend on the history of the early universe. We must distinguish between cosmological models which underwent a phase of cosmic inflation (of sufficient length for inflation to solve the

[21] This conclusion also depends on the fact that we have chosen a Calabi-Yau (or 6D base of our seven manifold) with only one 2-cycle on the 4-cycle Σ_4.

[22] One might worry at this stage that this is not the standard $M5_{\parallel}$ that we want. Recall however that at a generic point of the moduli space $M5_{\parallel}$ and $M5_{\perp}$ cannot be distinguished.

[23] Such $D8$ branes are allowed in massive type IIA theory. They correspond to $M9$-branes when lifted to M-theory [66]. One can reduce an $M9$ as either a nine-brane in type IIA theory or a $D8$-brane. The nine-brane configuration is exactly dual to the $E_8 \times E_8$ theory that we discussed before, where the required O9-plane comes from Gauss' law constraint. To avoid the orientifold of the nine-brane configuration in type IIA, we consider only $D8$-branes in type IIA.

[24] A fully compactified version would require a much more elaborate framework that we do not address here.

horizon problem of standard cosmology) and those which did not. Standard big bang cosmology, pre-big-bang cosmology [70], ekpyrotic cosmology [71], and string gas cosmology [72] are models in the latter class.

In models without inflation in which there was a very hot thermal stage in the very early universe all types of stable particles, strings, and branes will be present. Hence, in such models one expects all stable branes to be present. Since the wrapped $M2_\perp$-branes are stable but have too large a tension for the values of the parameters considered here, we conclude that there is a potential problem for our proposed magnetogenesis scenario without a period of inflation which would eliminate the $M2_\perp$-branes present in the hot early universe. However, if the temperature was never hot enough to thermally produce the $M2_\perp$-branes, as may well happen in string gas cosmology or in bouncing cosmologies, there would be no cosmological $M2_\perp$-brane problem.[25]

On the other hand, in inflationary universe scenarios, the number densities of all particles, strings, and branes present before the period of inflation was redshifted. To have any strings or branes present after inflation within our Hubble patch, these objects must be generated at the end of the period of inflation. Which objects are generated will depend critically on the details of the inflationary model. Since we are focusing on an M-theory realization of a particular heterotic string compactification, we will first discuss the issue of generation of cosmic superstrings in the context of a concrete realization of inflation in heterotic string theory due to Becker, Becker, and Krause [73]. In this model, several $M5$-branes are distributed along the $\frac{S^1}{\mathbb{Z}_2}$ interval. During the inflationary phase these are sent towards the boundaries by repulsive interactions. Slow-roll conditions are satisfied as long as the distance d between the $M5$-branes is much less than L the orbifold length. Once $d \sim L$ nonperturbative contributions which stabilize the orbifold length and Calabi-Yau volume at values consistent with a realistic value for G_N and a SUSY-breaking scale close to a TeV come into effect. This stabilization was used in the argument above and also leads to a small $M5_\parallel$ tension, so that while wrapped $M5$-branes will be produced at the end of inflation there is insufficient energy density to produce the $M2_\perp$-branes.

In our model, where cosmological evolution takes us to a generic point in the moduli space of G_2 structure manifolds (by rolling moduli), there may not be a problem with $M2_\perp$-branes—at least in the limit of compact G_2 structure manifolds with G_2 holonomy. This is because compact manifolds with G_2 holonomy have finite fundamental group. This implies vanishing of the first Betti number [74], which in turn means that $M2$-branes have no 1-cycles to wrap on. Once we make the G_2 manifolds noncompact (keeping the six-dimensional base compact with vanishing first Chern class[26]) we can still argue the nonexistence of finite 1-cycles, and therefore we do not expect a cosmological $M2$-brane problem.

VI. AMPLITUDE OF THE INDUCED SEED MAGNETIC FIELDS

Finally, we estimate the magnitude of the resulting seed magnetic fields, making use of the same arguments used in [2]. We want to calculate the magnetic field at a time t after decoupling in the matter-dominated epoch (specifically, at the beginning of the period of galaxy formation) at a distance r from the string. We will take this distance to be a typical galactic scale.

The magnetic field strength is given by

$$\begin{aligned} E_r &= c_+ r^{-1-(\alpha/\pi)} + c_- r^{-1+(\alpha/\pi)}, \\ B_\theta &= c_+ r^{-1-(\alpha/\pi)} - c_- r^{-1+(\alpha/\pi)}. \end{aligned} \tag{6.1}$$

The coefficients c_+ and c_- can be determined as in [22] by solving the anomalous Maxwell equations

$$dF = -\frac{\alpha}{\pi} d\left(\frac{a}{f}\right) \star F, \tag{6.2}$$

[see (5.2)] at the radius of the string core r_c given a string current with

$$\lambda = \frac{en}{2\pi}, \tag{6.3}$$

where n is the number per unit length of charge carriers on the string, all of which are moving relativistically. The result is [2,22]

$$B(r) \sim \frac{en}{2\pi}\left(\frac{r}{r_c}\right)^{\alpha\pi} r^{-1}. \tag{6.4}$$

Here a is a massless pseudoscalar Goldstone boson arising upon spontaneous symmetry breaking and α gives the strength of the coupling between the a and $F \wedge F$ (we borrow the terminology of [22]). There is no such coupling in our case, but we can still follow the analysis of [2], which we do here. During the formation of the string network at time t_c, the number density of charge carriers is of the order of T_c [where $T(t)$ is the temperature at the time t]:

$$n(t_c) \sim T_c. \tag{6.5}$$

As the correlation length $\xi(t)$ of the string network ex-

[25] Another way to get rid of the potential $M2_\perp$-brane problem might be to change the parameters of the model in order to reduce the $M2_\perp$-brane tension to an acceptable level.

[26] The base does not have to be a Calabi-Yau manifold to have vanishing first Chern class. See, for example, constructions in [59].

pands, the number density drops proportionally to the inverse correlation length. However, mergers of string loops onto the long strings lead to a buildup of charge on the long strings which can be modeled as a random walk [2] and partially cancel the dilution due to the expansion of the universe.[27] Taken together, this yields

$$n(t) \sim \left[\frac{\xi(t_c)}{\xi(t)}\right]^{1/2} n(t_c). \tag{6.6}$$

Assuming that the universe is dominated by radiation between t_c and t_{eq} and by matter from t_{eq} until t, we can express the ratio of correlation lengths in terms of ratios of temperatures [using $a(t) \sim T^{-1}$], with the result

$$n(t) \sim \left[\frac{T(t)}{T_{eq}}\right]^{3/4} \frac{T_{eq}}{T(t)} n(t_c). \tag{6.7}$$

Upon insertion of the above equations into (6.4) one finds

$$B(t) \sim \frac{e}{2\pi} \frac{T_{eq}}{r} \left[\frac{T(t)}{T_{eq}}\right]^{3/4} \left(\frac{r}{r_c}\right)^{\alpha\pi}. \tag{6.8}$$

By expressing the temperature in units of GeV and the radius in units of 1 m, and converting from natural units to physical units making use of the relation

$$\frac{e}{2\pi} \frac{\text{GeV}}{m} = 10^5 \text{ Gauss}, \tag{6.9}$$

we obtain

$$B(t) \sim 10^5 \text{ Gauss} \frac{T_{eq}}{\text{GeV}} r_M^{-1} \left[\frac{T(t)}{T_{eq}}\right]^{3/4} \left(\frac{r}{r_c}\right)^{\alpha\pi}, \tag{6.10}$$

where r_M is the radius in units of meters.

Evaluated at the time of recombination t_{rec} (shortly after the time t_{eq}) and at a radius of 1 pc, the physical length which turns into the current galaxy radius after expansion from t_{rec} to the current time, we obtain

$$B(t) \sim 10^{-20} \text{ Gauss} \left(\frac{r}{r_c}\right)^{\alpha\pi}. \tag{6.11}$$

Even with $\alpha = 0$ (our case), the value is of the same order of magnitude as is required to yield the seed magnetic field for an efficient galactic dynamo. If there were an anomalous coupling of our string to electromagnetism, the amplitude would be greatly enhanced since r_c is a microscopic scale whereas r is cosmological.

VII. DISCUSSION AND CONCLUSIONS

We have proposed a mechanism to generate seed magnetic fields which are coherent on galactic scales based on a M-theory realization of a particular heterotic string compactification. According to our proposal, wrapped $M5$-branes, which generically settle to a point in the moduli space of G_2 structure manifolds, act as superconducting cosmic strings from the point of view of our four-dimensional universe. These branes are stable, and carry charged zero modes which are excited via the Kibble mechanism in the early universe. Because of the scaling properties of cosmic string networks, the currents on the strings resulting from the charged zero modes generate magnetic fields which are coherent on the scale of the cosmic string network. This scale is proportional to the Hubble distance at late times, which means that the scale increases much faster in time than the physical length associated with a fixed comoving scale. It is this scaling which enables our mechanism to generate magnetic fields that are coherent on galactic scales at the time of galaxy formation.

Our setup is a possible string theoretic realization of the proposal made by Brandenberger and Zhang in [2]. The mechanism of [2] was based on pion strings which are unstable after the time of recombination [75], while the strings in our mechanism are stable. Thus, our current scenario predicts the existence of seed fields which are coherent on all cosmological scales, in contrast to the mechanism of [2] which admits a maximal coherence scale. This means our mechanism is in principle distinguishable from that of [2]. However, it is only seed fields on scales which undergo gravitational collapse which can be amplified by the galactic dynamo mechanism. The fields which we predict on larger scales will not have been amplified and thus will have a very small amplitude. These weak coherent fields are therefore a prediction of our setup, but their amplitude is presumably beyond our current detection abilities.

We have studied the viability of all branes arising in M-theory as sources of the superconducting cosmic strings required for our magnetic field generation mechanism. At a special point in the moduli space of G_2 structure manifolds where locally we have M-theory on $\frac{S^1}{\mathbb{Z}_2}$ fibered over a six-dimensional base, we can use tension and stability analyses to rule out all but the $M5_{\parallel}$-brane, as summarized in the table below (see [1] for details):

	Topology	Tension	Stability	Production
$M2_{\perp}$	√	×	√	×
$M2_{\parallel}$	×	⋯	⋯	⋯
$M5_{\perp}$	√	√	×	×
$M5_{\parallel}$	√	√	√	√

The wrapped $M5_{\parallel}$-brane in the $E_8 \times E_8$ heterotic theory realization compactified to $3 + 1$ dimensions avoids the instability pointed out by Witten [34]. Under cosmological evolution by rolling moduli, our system is driven to a

[27] Note that without string interactions, the correlation length $\xi(t)$ would not scale as t.

generic point in the moduli space of G_2 structure manifolds where we also expect nontrivial G fluxes evolving with time. At this point, under some reasonable assumptions, $M2_{\perp}$-branes cannot exist (no finite 1-cycles) and there is not much difference between $M5_{\perp}$ and $M5_{\parallel}$ branes. Thus for this $M5$-brane to be a valid candidate for producing primordial seed magnetic fields via the mechanism proposed in [2], we needed to verify that the brane can carry a superconducting current generated via charged zero modes at any generic point in the moduli space of G_2 structure manifolds. We have shown that this is indeed true. Thus, the wrapped $M5$-brane could supply the desired seed magnetic fields directly from string (or M) theory.

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