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LIMITS ON A STOCHASTIC GRAVITATIONAL WAVE BACKGROUND FROM OBSERVATIONS OF TERRESTRIAL AND SOLAR OSCILLATIONS

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ABSTRACT

Realistic models of the Earth and Sun are used to calculate their respective responses to a homogeneous, isotropic background of gravitational radiation. Solar velocity data constrain the energy density of such a background at a frequency of 4×10^{-4} Hz to be less than 10^2 times the closure density of the universe, as does Earth seismic data at frequencies of 2×10^{-3} Hz and 2×10^{-2} Hz. With improved data soon to be available it is likely that both of these limits will be lowered to below closure density. Errors in previous analyses of the Earth are pointed out.

Subject headings: cosmology — Earth: general — gravitation — relativity — Sun: general

I. INTRODUCTION

Previous analyses of the coupling of gravitational waves to the normal modes of the Earth have led to upper limits on a stochastic gravitational wave background (Forward *et al.* 1961; Weber 1967; Burke 1973; Zimmerman and Hellings 1980). The use of highly simplified models in addition to improper treatment of the coupling to high-frequency modes leads one to suspect these limits. In this paper we calculate the background level of gravitational radiation consistent with the most complete model of the Earth and seismic data that are available in the literature.

A similar calculation for the Sun is performed using a realistic solar model to determine vibrational eigenmodes. The long damping times predicted for low-frequency *p*- and *g*-modes make them especially interesting for detecting a stochastic background. The solar velocity data available for these modes are used to determine limits on the gravitational wave background which are comparable to those determined from Earth seismic data.

II. COUPLING OF GRAVITATIONAL WAVES TO A SPHERICALLY SYMMETRIC BODY

If the wavelength of a gravitational wave is large compared to the dimensions of a physical body (which is the case for systems discussed in this paper), the gravitational acceleration of an element of mass is given by the equation of geodesic deviation (Misner, Thorne, and Wheeler 1973) $d^2X_j/dt^2 = -\sum_k R_{j0k0} X_k$, where X_j are the coordinates of the element relative to the center of mass and R_{j0k0} is the Riemann curvature tensor evaluated at the center of mass. For a plane wave of angular frequency ω and propagating in the $\hat{y}$ direction it is straightforward to show that the effective force density, when expressed in spherical coordinates, has the form

$$f(r, \theta, \phi, t) = \frac{\rho(r)\omega^2}{4} e^{i\omega t} \times \nabla\left[\!\left[r^2 \left\{ \left[\left(\frac{2\pi}{15}\right)^{1/2} (Y_{22} + Y_{2-2}) - \left(\frac{4\pi}{5}\right)^{1/2} Y_{20} \right] h_+ + \left(\frac{8\pi}{15}\right)^{1/2} (Y_{2-1} - Y_{21}) h_\times \right\} \right]\!\right], \quad (1)$$

where the Y_{2m} are normalized spherical harmonics, $\rho(r)$ is the mass density of the body, and h_+ and $h_\times$ are the dimensionless amplitudes of the two polarizations of the wave (Misner, Thorne, and Wheeler 1973). This force density may be expressed as a linear combination of vector spherical harmonics (Edmonds 1960)

$$f = \sum_{m=-2}^{2} a_m(r) \mathbf{Y}_{21m} e^{i\omega t} , \quad (2)$$

where, for example, $a_0(r) = -(\pi/2)^{1/2}\omega^2 h_+ r\rho(r)$.

The eigenfunctions of the oscillations of Earth and Sun may also be expressed in terms of combinations of vector spherical harmonics. In particular, consider eigenfunctions of the form

$$\xi_{nlm}(r, \theta, \phi) = U_{nlm}(r) Y_{lm} \hat{r} + V_{nlm}(r) \nabla Y_{lm} , \quad (3a)$$

or

$$\xi_{nlm}(r, \theta, \phi) = a_{nlm}(r) \mathbf{Y}_{ll+1m} + b_{nlm}(r) \mathbf{Y}_{ll-1m} . \quad (3b)$$

Toroidal modes are not described by (3a), but these modes do not possess time-dependent quadrupole moments and therefore do not couple to gravitational waves.

In the presence of a gravitational wave, the displacement amplitude of the body can be expressed as

$$\xi(r, \theta, \phi, t) = \sum_{nlm} C_{nlm}(t) \xi_{nlm}(r, \theta, \phi) . \quad (4)$$

Substituting (4) into the equation of motion with a force density given by (1) will yield equations of the form

$$\frac{d^2 C_{nlm}}{dt^2} + \frac{2}{\tau_{nlm}} \frac{dC_{nlm}}{dt} + \omega_{nlm}{}^2 C_{nlm} = f_{nlm} e^{i\omega t} , \quad (5)$$

for small, lightly damped oscillations. The quantities ξ_{nlm}, τ_{nlm}, and ω_{nlm} are calculated from knowledge of the mechanical properties of the Earth (or Sun) and

$$f_{nlm} = \int d^3r \xi_{nlm} \cdot f \Big/ \int d^3r \xi_{nlm} \cdot \xi_{nlm} \rho(r) , \quad (6)$$

are the projections of the gravitational wave force on the eigenmodes. The orthogonality of vector spherical harmonics and equations (2) and (3b) show explicitly that only modes with $l = 2$ couple to gravitational waves.

We assume spherically symmetric models for Earth and sun. Since these systems are invariant under a rotation about any axis the constants τ_{nlm} and $\omega_{nlm}{}^2$ of equation (5) and the coefficients U_{nlm} and V_{nlm} of equation (3a) must be independent of m.

For an unpolarized isotropic background, the mean square dimensionless gravitational wave amplitudes for waves propagating in a frequency interval $d\omega$ and a solid angle $d\Omega$ are

$$\langle h_+{}^2 \rangle = \langle h_\times{}^2 \rangle = h^2(\omega) d\omega d\Omega \,, \tag{7}$$

where $h^2(\omega)$ is the intensity of h and is related to the spectral energy density $S_E(\omega)$ of the waves by (Misner, Thorne, and Wheeler 1973)

$$S_E(\omega) = \frac{c^2}{2G} \omega^2 h^2(\omega) \,, \tag{8}$$

where c is the velocity of light and G the gravitational constant.

The mean square mode coefficient of the n, m normal mode due to waves propagating in the y-direction is, from equations (2), (3), (5), (6), and (7):

$$\langle C_{n2m}{}^2 \rangle = \int_{-\infty}^{\infty} \frac{f_{n2}{}^2(\omega) d\omega}{(\omega_{n2}{}^2 - \omega^2)^2 + 4\omega^2/\tau_{n2}{}^2} = \frac{S_{n2}{}^2 \alpha_m{}^2 \omega_{n2}{}^2 h^2(\omega_{n2}) \tau_{n2} \, d\Omega}{16} \,, \tag{9}$$

where $\alpha_0 = -(4\pi/5)^{1/2}$, $\alpha_1 = -(8\pi/15)^{1/2}$, $\alpha_2 = -(2\pi/15)^{1/2}$, and $\alpha_m = (-1)^m \alpha_{-m}$; and S_{n2} depends only on integrals of $U_{n2}(r)$, $V_{n2}(r)$, and $\rho(r)$. Because of spherical symmetry, the response of the system to waves propagating in other directions is obtained by applying rotations to the above solution. Integrating over all directions gives a mean square coefficient due to an isotropic background of

$$\langle C_{n2m}{}^2 \rangle = \frac{16\pi^3}{75} S_{n2}{}^2 \omega_{n2}{}^2 h^2(\omega_{n2}) \tau_{n2} \,, \tag{10}$$

which is independent of m as required by spherical symmetry and isotropy.

In order to compare with observations, $\langle C_{n2m}{}^2 \rangle$ must be expressed in terms of the power spectral density of the observed displacement, velocity, or acceleration of the surface of the Earth or Sun.

III. LIMITS IMPLIED BY SOLAR OBSERVATIONS

Speculation that an observed solar normal mode is excited by nearly monochromatic gravitational waves (Walgate 1983) is unwarranted (Kuhn and Boughn 1984). However, interesting limits on the background flux can be obtained from observed upper limits to certain mode amplitudes. While observational data on solar normal-mode amplitudes are much worse than their terrestrial counterparts, the large mass of the Sun and expected large mechanical Q's ($>10^9$ for some modes) make the Sun an interesting detector of a stochastic background of gravitational waves.

To infer the flux with any certainty, of course, requires a good model of the solar interior. It is beyond the scope of this paper to critique the relative merits of different solar models. Bahcall *et al.* (1982) discuss the uncertainty in most of the model parameters in the context of the solar neutrino problem. The model used in this paper is similar to their "standard solar models" (e.g., a central density of 147 g cm^{-3}, mixing length to scale height ratio $l/H = 1.5$, $Z = 0.02$, and $Y = 0.24$). Using this model, we find solar p-mode eigenfrequencies that agree with observed frequencies in the 5-minute band to a few parts in 1000.

In the simplest form the adiabatic eigenmodes are found by solving for the Lagrangian displacement field, ξ^*, from

$$\frac{\partial^2 \xi^*}{\partial t^2} + \mathscr{L}(\xi^*) = \frac{f(r, \theta, \phi, t)}{\rho(r)} \,, \tag{11}$$

where $\mathscr{L}$ is a Hermitian differential operator (Cox 1980), with eigenmodes $\xi_{nlm}{}^*$ such that

$$\mathscr{L}(\xi_{nlm}{}^*) = \omega_{nl}{}^2 \xi_{nlm}{}^* \,. \tag{12}$$

To a high degree of approximation the nonadiabatic eigenmodes described below also satisfy equation (12). Thus we take $\xi_{nlm}{}^* = \xi_{nlm}$ to obtain modal equations of the form (5) and τ_{nl} to be the reciprocal imaginary part of the complex eigenvalue found from the nonadiabatic analysis.

We use a Henyey type oscillation code (the Sacramento Peak Observatory pulsation code) similar to that used by Saio and Cox (1980). Nonadiabatic eigenfunctions and complex eigenvalues are obtained from the linearized equations described above.

It is unlikely that errors in the calculated eigenmodes or eigenfrequencies will be large enough to affect our conclusions. The damping times are more uncertain, and, for the high-frequency p-modes, are at best only estimates. The low-order mode damping times should be much less affected by problems with the treatment of radiative transfer near the photosphere, since the mode energy there is much smaller than it is below the convection zone, where the diffusion approximation works well (cf. Cox 1980).

Modes with periods near 5 minutes are observed in the solar spectrum (cf. Duvall and Harvey 1983) and are apparently overstable. The excitation mechanism is not well understood, although it is likely that a clear understanding of convection and radiative transfer (cf. Goldreich and Keely 1977; Christensen-Dalsgaard and Frandsen 1983) near the photosphere will elucidate the problem. Lacking such a model, we cannot attribute the excitation of modes in this frequency range to gravitational waves.

Table 1 lists a representative set of mode frequencies, damping times, and overlap coefficients, S_{n2}. These coefficients are the same factors that enter equation (10) and are calculated from

$$S_{n2} = \frac{\int \rho(r) r^3 U_{n2}(r) dr + 3 \int \rho(r) r^3 V_{n2}(r) dr}{\int \rho(r) r^2 [U_{n2}(r)]^2 dr + 6 \int \rho(r) r^2 [V_{n2}(r)]^2 r} \,. \tag{13}$$

Possibly the best observational limits on mode amplitudes are contained in the Birmingham group's data (cf. Isaak 1981). They observe line-of-sight velocity integrated over the full disk of the Sun. The observed velocity is approximately the intensity-weighted line-of-sight component of the modal velocity. Christensen-Dalsgaard and Gough (1982) have calculated these modal response functions for several modes. Adopting the ratio of transverse to vertical velocities of the $l = 2$ g-mode near a period of 160 minutes, we obtain an expression for the mean squared observed velocity:

$$\langle v^2 \rangle = \langle C_{n2m}{}^2 \rangle \omega_{n2}{}^2 R_\odot{}^2 K^2 \,, \tag{14}$$

where $K^2 = 0.4$ is approximately true for the low-order $l = 2$ modes. The additional factor of the solar radius squared $R_\odot{}^2$

TABLE 1

SOLAR MODES

$\omega_n[10^{-3}\ \mathrm{s}^{-1}]$	Period [min]	τ_n[s]	S_n	Flux [erg s^{-1} cm^{-2} Hz^{-1}]
g modes:				
0.665	157	1.5×10^{12}	0.017	1.1×10^{15}
1.18	88	5.3×10^{12}	0.43	1.4×10^{11}
1.36	77	8.3×10^{13}	0.87	1.7×10^{9}
p modes:				
2.40	44	4.7×10^{12}	−6.2	1.8×10^{8}
4.18	25	3.9×10^{10}	−8.4	4.0×10^{9}
13.3	7.9	1.8×10^{6}	−11.9	4.4×10^{12}

enters because the radial functions $U_{n2}(r)$ and $V_{n2}(r)$ are normalized to have the value 1 $R_\odot$ at the surface. C_{n2m} are the dimensionless normal-mode coefficients which after averaging are independent of m (see eq. [10]).

Combining equations (8), (10), and (14) gives the background energy density in terms of the observed velocity,

$$S_E(\omega) = \frac{75c^2}{32\pi^3 G R_\odot^2} \frac{\langle v^2 \rangle}{\omega_{n2}^2 K^2 \tau_{n2} S_{n2}^2} . \quad (15)$$

Van der Raay and Isaak (1984) have suggested that the velocity noise is between 1 and 4 cm s^{-1} near periods of 44 minutes. Since the spectral resolution of the data is much worse than $1/\tau_{n2}$ for these periods and since we do not know the actual velocity power probability distribution in the data bins corresponding to the expected normal mode frequencies, we will adopt $\langle v^2 \rangle = 1$ cm^2 s^{-2} as a reasonably conservative mean squared velocity limit on the gravitational wave background. With additional information on the observed power spectrum, a precise statistical statement of the gravitational wave limits could be made. We believe the limits derived from these solar data yield roughly 1 σ confidence level limits on the gravitational flux. Table 1 shows the corresponding energy density limits for the modes listed.

IV. LIMITS IMPLIED BY TERRESTRIAL OBSERVATIONS

Although the Earth is much smaller than the Sun, its relatively small cross section is compensated for by our ability to make more sensitive measurements. As was pointed out by Weber (1967), terrestrial seismic data may place interesting limits on a gravitational wave background. Several relatively crude calculations have been performed to estimate the terrestrial response to gravitational waves (Forward *et al.* 1961; Weber 1967; Zimmerman and Hellings 1980). Given the advanced state of geophysical models for, at least, the spherically symmetric Earth, a realistic calculation is warranted.

The Earth model we use has been derived from terrestrial normal mode observations (model 1066A, Gilbert and Dziewonski 1975). It is believed that the density and elastic constants used in this model are correct to within a few percent throughout the Earth (Jordan 1980). In the following calculations we use eigenmodes originally calculated by R. Buland. A description of the calculational technique is contained in Backus and Gilbert (1968). The mechanical Q's of these modes are less well determined (cf. Sailor and Dziewonski 1978; Masters and Gilbert 1983). Measured Q-factors for the modes of interest are too sparse to be used directly, but knowledge of the eigenfunctions and local mechanical properties of the Earth model is sufficient to calculate the Q for each mode (cf. Backus and Gilbert 1968; Dahlen 1980). We use the six-shell model with no bulk dissipation (shear only) described by Masters and Gilbert (1983). This calculation yields errors in Q of 5–30% with the observed modes from which it was derived.

Unlike the solar velocity observations, the terrestrial seismic data are a direct measurement of the acceleration of the surface of the Earth with respect to a local inertial frame. An accelerometer responds to: (1) the surface acceleration, (2) the gravitational field due to the redistribution of mass of a normal mode, (3) the displacement of the accelerometer in the background gravitational field. Gilbert (1980) has explicitly calculated how these effects contribute to the vertical and horizontal accelerometer response. For a mode of the form given by equations (3a) and (4) the vertical acceleration is $a_0 = K_n \omega_{n2}^2 C_{n2m}$, where $K_0 = 0.78$ for the lowest ($n = 0$) $l = 2$ mode and approaches 0.63 as n increases.

In addition we must consider the effect of the background gravitational wave on the accelerometer itself. In the absence of the Earth, an accelerometer is in a local inertial frame even in the presence of a gravitational wave and so records no acceleration. Its reponse on the surface of the Earth is thus the difference between a_0 discussed above and the free acceleration due to the gravitational wave. As we will see below, this effect is important when considering the higher frequency modes and especially the "off-resonance" response of the Earth. The analog of this effect in the solar data, namely the direct interaction of the gravitational wave with the telescope and observer, is negligible because the Q's of the solar oscillations are so large.

The response of the Earth to an excitation at one of the normal mode frequencies can be expressed as the sum of the resonance response of that mode and the off-resonance response of all the other modes. For the lowest frequencies the resonant term dominates by a large factor, and one can ignore the off-resonance response. At higher frequencies the resonance response can be smaller than the sum of the off-resonance terms. However, at these frequencies the off-resonance component is to a good approximation equal to, and therefore cancels, the free response of the accelerometer, as discussed above. As a consequence, even at high frequencies one need consider only the resonant excitation.

The mean square displacement amplitude of a seismometer is, analogous to equation (14),

$$\langle X^2 \rangle = \langle C_{n2}^2 \rangle R_E^2 K_n^2 , \quad (16)$$

where R_E is the radius of Earth and K_n lies between 0.63 and 0.78 as discussed above. As before, it follows from equations (8), (10), and (16) that the energy density of the gravitational wave background is given by

$$S_E(\omega) = \frac{75c^2}{32\pi^3 G R_E^2} \frac{\langle X^2 \rangle}{K_n^2 \tau_n S_{n2}^2} . \quad (17)$$

TABLE 2

TERRESTRIAL MODES

n	$\omega_n[\times 10^{-2}\ s^{-1}]$	Period [min]	τ_n[s]	S_n	Flux [erg s^{-1} cm^{-2} Hz^{-1}]
0	0.194	53.9	6.8×10^5	0.70	7.1×10^9
2	0.599	17.5	1.0×10^6	9.7×10^{-4}	1.6×10^{12}
4	1.08	9.7	1.0×10^5	8.7×10^{-2}	5.8×10^7
22	5.16	2.0	6.1×10^4	1.7×10^{-3}	1.7×10^8
28	6.35	1.6	1.2×10^5	4.8×10^{-5}	4.4×10^{10}
60	13.48	0.78	1.8×10^4	-3.7×10^{-3}	3.4×10^6

The seismic background noise is apparently limited by atmospheric pressure load fluctuations and ocean wave action (cf. Murphy and Savino 1975) and shows a strong frequency dependence. We have used the mean vertical seismic power density described by Agnew and Berger (1978) to calculate the following limits. Since the spectral resolution of the observed spectra is larger than the resonant widths of the Earth modes, the mean square displacement is given by $\langle X^2 \rangle = P(\nu_n)\Delta\nu$, where $P(\nu_n)$ is the vertical displacement noise power at frequency ν_n and $\Delta\nu$ is the spectral resolution. Furthermore, since the power spectra of Agnew and Berger are averages of between 20 and 30 individual spectra, the associated noise on the spectra is approximately a factor of 5 below the average power. As no excitations of Earth modes are seen at this level, we obtain roughly a 1 σ limit on the mean square displacement of a given mode of $\langle X^2 \rangle < P(\nu_n)\Delta\nu/5$. Table 2 shows the eigenfrequencies, Q's, and corresponding 1 σ background gravitational wave energy density limits for some $l = 2$ Earth modes.

Zimmerman and Hellings (1980) computed the response of an accelerometer to gravitational wave excitation at frequencies between those of the Earth's normal modes, i.e., the off-resonance response. Primarily because they did not take into account the direct action of the gravitational wave on the accelerometer, the limits they derived for this type of excitation were too low by four orders of magnitude.

In Figure 1 we have plotted the resonance and off-resonance background energy limits. The off-resonance limit was calculated from the approximate expression

$$S_E(\omega) = \frac{75}{64\pi^2} \frac{c^3}{G} \frac{S_a(\omega)}{R^2\omega^2K^2} \left[\sum_n \frac{S_{n2}\,\omega_{n2}{}^2}{(\omega_{n2}{}^2 - \omega^2)} \right]^{-2} , \quad (18)$$

where $S_a(\omega)$ is the spectral density of the surface acceleration and the sum extends over the first 61 $l = 2$ Earth modes. This expression includes the direct response of the accelerometer to gravitational waves.

V. DISCUSSION

Upper limits on the flux density of a homogeneous isotropic background of gravitational radiation have been determined from terrestrial and solar observations and are illustrated in Figure 2. For comparison, the limits deduced from pulsar timing (Romani and Taylor 1983) and from the Stanford cryogenic gravitational wave detector (Boughn *et al.* 1982) are also plotted.

The limit labeled "Earth normal modes" is from one to three orders of magnitude higher than that of Zimmerman and Hellings (1980) and Weber (1967) due to our more realistic model of the Earth. The limit labeled "off-resonance response" derived by considering off-resonance excitation is about four orders of magnitude larger than the analogous limits of Zimmerman and Hellings as described above.

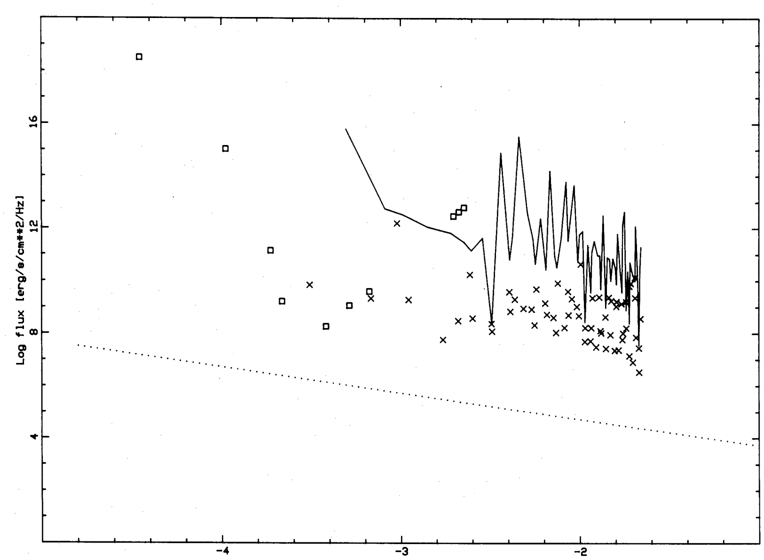

FIG. 1.—Background gravitational radiation flux limits. Squares represent limits derived from solar observations, crosses come from resonance seismic data, and the solid line is derived by considering off-resonance excitation. The dotted line shows the background flux in one octave which is equal to closure density ($H = 100$ km s^{-1} Mpc^{-1}).

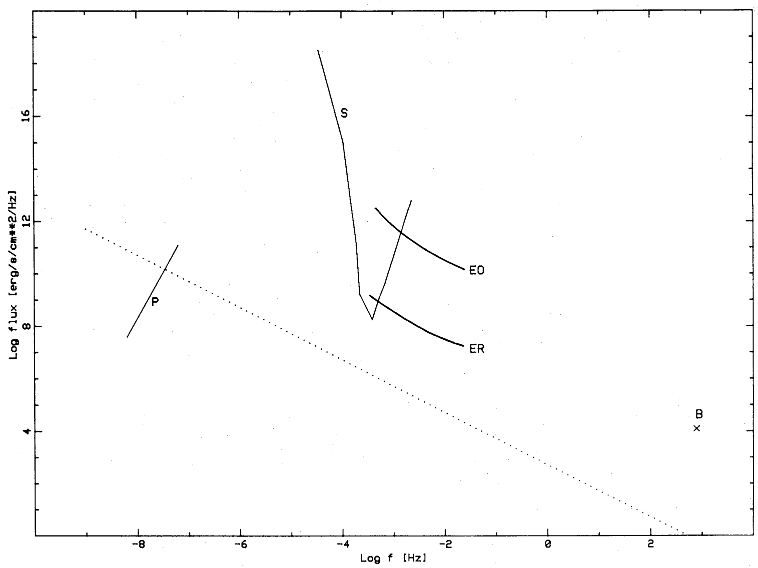

FIG. 2.—Background gravitational flux limits. The dotted line is the same as for Fig. 1. Symbols P, S, and B indicate, respectively, limits obtained from pulsar timing data, the solar observations, and laboratory bar detectors. ER and EO show the seismic resonance and off-resonance flux limits.

A useful benchmark with which to compare these limits is the closure density of the universe, i.e., $\rho_c = 2 \times 10^{-29}$ g cm^{-3} (assuming a Hubble constant of 100 km s^{-1} Mpc^{-1}). The curve labeled "closure density" in the figure is the flux of gravitation radiation, F_ν, such that $\nu F_\nu = \rho_c c^3$; i.e., the closure density is contained in one octave. Any gravitational wave background substantially larger than this value conflicts with observations which imply the universe is approximately open. Carr (1980) discusses various scenarios for the generation of gravitational radiation and concludes that νF_ν could be as large as $10^{-2} \rho_c c^3$ over periods in the range 10^{-3} s to 10^5 s. A more stringent requirement, $\nu F_\nu < 10^{-4} \rho_c c^3$, is imposed on a primordial gravitational wave background in order to be consistent with cosmological nucleosynthesis (Carr 1980).

It is clear that the limits derived in this paper do not constrain any models. The "Earth mode" limits, however, are due to the finite spectral resolution of the Agnew and Berger (1978) data and may decrease by two orders of magnitude or more if one would observe for long periods of time at a seismically quiet location. The "solar mode" limits are also due to finite signal-to-noise and may decrease by orders of magnitude in the near future. If this is the case, the solar limits will approach the level of some of the scenarios of Carr (1980) and the Sun may prove to be one of the most sensitive detectors of background gravitational radiation at milliherz frequencies.

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