1973

Fine-Scale Anisotropy of the Microwave Background: An Upper Limit at lambda=3.5 Millimeters

Paul E. Boynton

Bruce Partridge
Haverford College, bpartrid@haverford.edu

1973

Fine-Scale Anisotropy of the Microwave Background: An Upper Limit at lambda=3.5 Millimeters

Paul E. Boynton

R. Bruce Partridge
Haverford College

Repository Citation
(with P. Boynton) Fine-scale Anisotropy in the Cosmic Background Radiation at l = 3.5 mm, Astrophysical Journal, 181, 243, 1973.

THE ASTROPHYSICAL JOURNAL, 181:243–253, 1973 April 15

FINE-SCALE ANISOTROPY OF THE MICROWAVE BACKGROUND: AN UPPER LIMIT AT $\lambda = 3.5$ MILLIMETERS*

PAUL E. BOYNTON
University of Washington, Seattle

AND

R. B. PARTRIDGE†
Haverford College, Haverford, Pennsylvania
Received 1972 March 8; revised 1972 September 29

ABSTRACT

Using the 36-foot NRAO telescope[1] at $\lambda = 3.5$ mm, we have set an upper limit of 0.0043° K, with 90 percent confidence, on the fluctuations in the cosmic microwave background. The angular scale of the measurement was $\sim 80''$. If discrete sources produce all of the microwave background, their number must exceed ~ 0.35 Mpc^{-3}.

Subject headings: cosmic background radiation — cosmology — radio radiation

I. INTRODUCTION

If the primeval-fireball model of Dicke *et al.* (1965) is the correct explanation for the cosmic microwave background, then the background radiation ought to be essentially isotropic on all angular scales. However, small anisotropies—not small in angular scale, but small in the sense that variations in the intensity ΔI are much less than I—are possible if the expansion of the Universe is anisotropic (Thorne 1967; Rasband 1971) or if the velocity of the solar system with respect to the comoving coordinate system is nonzero (Peebles and Wilkinson 1968). On a smaller angular scale (less than or about 30°) anisotropy may be produced by large density inhomogeneities (Rees and Sciama 1968; Sunyaev and Zel'dovich 1970), by motions of protogalactic plasma (Chibisov and Ozernoy 1969), or even by long-wavelength gravitational waves (Dautcourt 1969). Fractional variations in the intensity of the background radiation, $\Delta I/I$, are expected to be $\lesssim 10^{-3}$ for all the above.

In the past few years, however, several alternative models for the background radiation have been suggested (e.g., Wolfe and Burbidge 1969; Wagoner 1969). Most of them employ individual sources rather than the primeval fireball to produce the radiation. What is observed, in this picture, is the sum of the radiation produced by many sources at all redshifts. The most detailed version of such a model is that of Wolfe and Burbidge (1969). They show that sources with a narrow range of spectral indices and intensities can reproduce the observed spectrum of the radiation. However, they do not consider adequately a second test of such models, namely, the small-scale fluctuations in the intensity of the radiation (see Salpeter and Hazard 1969).

This question has been treated more generally by Smith and Partridge (1970). They consider a situation in which a series of radiometric measurements is made with an antenna beam of solid angle Ω at different points of the sky. They then show that

* Research supported in part by the National Science Foundation while the authors were at Princeton University.

† Alfred P. Sloan Research Fellow.

[1] The National Radio Astronomy Observatory is operated by Associated Universities, Inc., under contract with the National Science Foundation.

fluctuations in the intensity of the background radiation, ΔI, will result from statistical fluctuations in the number of sources in Ω at different antenna positions. The results are presented in the form of a lower limit on the value of a parameter

$$\mu = N\Omega\left(\frac{\Delta I}{I}\right)^2 \propto \frac{\text{number of sources per volume}}{\text{number of sources per solid angle}} . \quad (1)$$

for several cosmological models and several limiting redshifts. We have altered their notation, and use N for the present number density of sources per Mpc^3. Also, Ω is measured in (arc min)^2, and is given by

$$\Omega = \frac{(\int\int G(x, y)dxdy)^2}{\int\int G^2(x, y)dxdy} ,$$

where $G(x, y)$ is the gain of the antenna employed.

Measurements to date have been made *either* with high sensitivity (implying a low upper limit on $\Delta I/I$), but with rather large beamwidths ($\geqslant$ 10 arc min) (Conklin and Bracewell 1967; Parijskij and Pyatunina 1970), *or* with tighter beams but poorer sensitivity (Epstein 1967; Penzias, Schraml, and Wilson 1969). The newest and most useful of these measurements is that of Parijskij and Pyatunina (1970) who worked with a 1.'4 × 20' fan beam at $\lambda = 3.95$ cm. Their value for $\Delta I/I$ on this angular scale is 2.6×10^{-4}, with no confidence level given.

In this paper, we report the results of a measurement of the microwave background on an angular scale of $\sim$80" at $\lambda = 3.5$ mm. We elected to work at a much shorter wavelength than Parijskij and Pyatunina for two reasons. First, we could obtain much higher angular resolution at 3.5 mm (at the cost of increased receiver and atmospheric noise). Decreasing Ω provides a more sensitive test of the discrete source model (see eq. [1]). Second, as may be seen from the work of Wolfe and Burbidge (1969) or Wagoner (1969), observations near the peak of the putative blackbody spectrum provide a particularly sensitive test of the discrete-source models. The discrete-source models predict that the short-wavelength radiation will come predominantly from relatively nearby sources, with redshifts $z \leqslant 2$. Assuming a uniform density of sources in comoving coordinates, the number of sources per unit solid angle is less for smaller z. For discrete source models, we thus expect larger intensity fluctuations at short wavelengths than at long wavelengths.[2] In addition, at millimeter wavelengths, local (galactic) sources which might make a spurious contribution to ΔI are less intense than at longer wavelengths.

In the remainder of this paper, we first discuss our experimental technique, then the treatment of the data. Some related measurements of a nearby galaxy (M31) are described, and a number of conclusions are drawn from our results.

II. THE MEASUREMENTS

For these observations, we used the 85-GHz receiver at the focus of the NRAO 36-foot (11-m) dish at Kitt Peak, Arizona. The main antenna beam was 80" by 72", where both figures are full widths at half-maximum. Assuming a Gaussian profile for the main beam, we find

$$\Omega = 3.6\,(\text{arc min})^2 . \quad (2)$$

At the time we were using it, the receiver had a system noise temperature of $\sim$2500° K, and the atmospheric contribution to the antenna temperature varied between 20° and 30° K depending on the zenith angle. The bandpass of the receiver was 1 GHz.

[2] Intensity fluctuations produced by density inhomogeneities, gravity waves, or other causes *in the fireball model* are expected to be independent of the wavelength of observation.

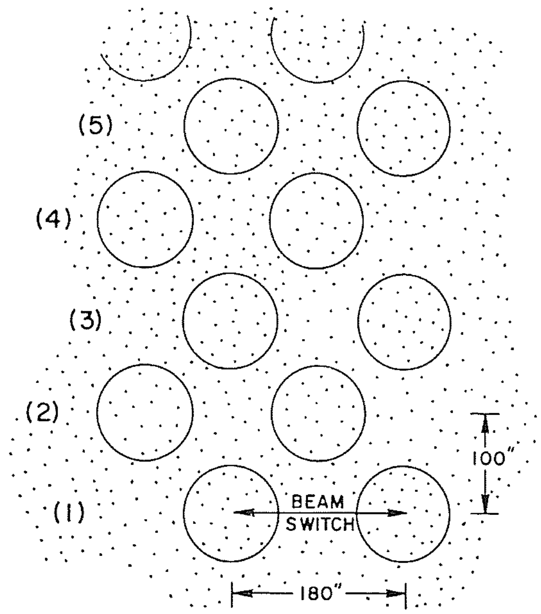

FIG. 1.—Sky sampling pattern

Our method of making measurements is indicated schematically in figure 1. We operated the receiver in the beam-switch mode, so that the output was proportional to the difference in antenna temperature between a pair of approximately N-S patches in the sky, separated by 180". Every half-hour we stepped the antenna 100" in right ascension and 90" in declination to move to a new pair of patches. Up to 12 such pairs were observed in each of four regions of the sky: the approximate coordinates of these four regions are given in table 1. A declination of +32° was chosen so that the regions passed through the local zenith. The right ascensions were selected to ensure that one of the four regions was always within 45° of the zenith during the hours in which we could make observations (roughly 3:00 P.M. through 9:00 A.M. local time). Two of the regions (numbers 100 and 200) lay near the north galactic pole, where we expect to have the minimum interference from possible microwave sources in the galaxy. On the other hand, region 400 lay in the galactic plane, roughly in the direction of the anticenter.

Ideally, in making these observations, we would have used a much shorter integration time than ½ hour on each pair of patches, and then repeated the measurements

TABLE 1

COORDINATES OF THE REGIONS SCANNED

Region Number	α (1970)	δ (1970)
100..................	11^h30^m	+32°
200..................	13 00	+32
300..................	01 45	+32
400..................	06 00	+32

NOTE.—In each region, an area ~5′ × 20′ was scanned.

several times each night. This was not possible because the beam switch at the 36-foot telescope at the time we used it operated solely in elevation or azimuth, but not in declination (the mechanism for rotating the receiver box was inoperative). In celestial coordinates, the small 180″ angle through which the beam switched thus depended on the hour angle. Hence a single pair of patches had to be observed at the same hour angle each day if data from different days were to be combined. As a result, we could make only one observation of each pair of patches each day: to reduce the noise sufficiently, we required the half-hour integration time. During this interval, in the worst case, the antenna pattern was displaced by less than 10″.

For most of our runs, the beam-switched output was digitally integrated and recorded every 50 seconds. Thus for each pair of patches observed for one half-hour on a single night we had 30 to 35 independent measurements.

Calibration was provided by standard "ON-OFF" observations of Saturn and Jupiter. The values obtained were

$$\text{Saturn:}\quad 0.0183^\circ \pm 0.0003^\circ\ \text{K per count per second},$$

$$\text{Jupiter:}\quad 0.0185^\circ \pm 0.001^\circ\ \text{K per count per second}.$$

We have employed the value for Saturn in subsequent calculations.

III. DATA ANALYSIS

The basic data may be arranged in four tables (one for each region of the sky) with 12 columns (for a maximum of 12 patch-pairs sampled in each sky region) and 12 rows (for a maximum of 12 runs made on each region). The table entries are sets of 30 to 35 independent consecutive integrations of the receiver output on each patch-pair. A small portion (the second run on the third and fourth patches) of the table entries for region 100 is shown in table 2. The four tables are not complete. Regions 200 and 300 were ravaged by the realities of daytime observations; thus the tables for regions 100 and 400 contain nearly all the information.

Each independent integration, X, is a measure of the difference in intensity between the two patches constituting a pair, masked of course by both the statistical fluctuations in receiver output and systematic effects:

$$X = (A + G) - (1 - \epsilon)(A' + G) + S + L .$$

Here ϵ represents the asymmetry of the system response in the two feed-horn positions corresponding to the two patches of a pair with incident microwave intensities A and A'. G is the background, primarily atmospheric radiation, but with a measurable contribution from ground radiation into the feed-horn side lobes. S characterizes the stochastic noise components, and the final term includes occasional abrupt changes in the receiver output level characteristic of wave-guide systems subject to changing mechanical stresses.

X may be rewritten as

$$X = D + \epsilon G + S + L ,$$

where

$$D = A - A' + \epsilon A'$$

and

$$\text{var}\, D = 2\, \text{var}\, A = 2\sigma_{\text{sky}}^2 .$$

The difference D contains the desired information about "sky" roughness, σ_{sky}. Ideally, in the absence of beam-switch asymmetry ($\epsilon = 0$) and stress-related changes in

TABLE 2

SAMPLE DATA FROM RUN 2, REGION 100*

Pair 3		Pair 4	
Time	Signal	Time	Signal
0.00	522.58	33.85	524.16
1.00	523.36	34.85	510.84
2.00	522.78	35.85	514.70
3.00	526.12	36.85	512.62
4.00	534.64	37.85	518.84
5.00	522.92	38.85	514.00
6.00	526.96	39.85	509.46
7.00	525.88	40.85	514.74
8.00	516.06	41.85	514.36
9.00	526.42	42.85	517.38
10.00	519.08	43.85	515.18
11.00	521.98	44.85	514.40
12.00	518.82	45.85	515.62
13.00	521.36	46.85	515.70
14.00	519.54	47.85	516.58
15.00	520.86	48.85	513.34
16.00	522.40	49.85	517.34
17.00	513.44	50.85	512.04
18.00	523.44	51.85	511.40
19.00	520.08	52.85	512.94
20.00	526.34	53.85	514.62
21.00	518.52	54.85	519.48
22.00	519.60	55.85	512.40
23.00	520.70	56.85	516.78
24.00	518.80	57.85	513.28
25.00	515.50	58.85	513.16
26.00	521.16	59.85	512.12
27.00	520.06	60.85	511.98
28.00	518.78	61.85	513.94
29.00	517.96	62.85	505.60
30.00	518.86	63.85	503.24
31.00	515.90	64.85	507.84
32.00	518.06	65.85	509.88
33.00	518.94	66.85	513.64

* Elements (2, 3), (2, 4) of the 12 × 12 data table for region 100—i.e., run 2, patch pairs 3, 4. Time is in units of 51 seconds (50-s integration and 1-dead time), with the origin arbitrarily selected at the beginning of the integrations on patch pair 3.

the receiver output (L = constant), the table of integrations, suitably averaged over runs (rows), would yield a column-to-column variance which contains the sky roughness; but it would also contain a variance component, $\sigma_{\rm rec}^2$ (inversely proportional to the integration time) due to receiver noise.

A similar analysis, sampling row-to-row differences, would yield the intrinsic receiver-noise level, $\sigma_{\rm rec}^2$, exclusive of any sky variation. A comparison of "column variance," $\sigma_{\rm sky}^2 + \sigma_{\rm rec}^2$, to "row variance," $\sigma_{\rm rec}^2$, could then determine, through methods of statistical inference, an upper limit for $\sigma_{\rm sky}^2$ at some predetermined confidence level, say 90 percent.

This procedure was carried out, but not without difficulty. Unfortunately a defective feed-horn assembly produced a response asymmetry ϵ (actually a reflection coefficient asymmetry associated with beam switching) of a few percent. This difficulty led to a secant trend in zenith angle as large as a few tenths of a degree Kelvin arising from atmospheric emission. We also noted a 0.1° K contribution periodic in zenith angle

(~20° period) which we ascribe to spillover of the feed-horn sidelobes. Both these effects can be seen in all four regions with nearly the same amplitude and zenith-angle symmetry, and thus are clearly instrumental. In addition, there was an abrupt 2° K signal-level change which occurred each time the dish was slewed in azimuth to cross the zenith. These effects are up to two orders of magnitude larger than the receiver-noise contribution which should ultimately have limited the observability of sky roughness.

Several column-to-column differencing schemes were employed in an attempt to eliminate the angle-dependent (or, equivalently, time-dependent) background effects. Each failed because the background resulting from switching asymmetry varied significantly, even though slowly, over the 30 or so integrations on a given patch-pair. This latter fact forces the use of some form of curve-fitting to the basic data to remove as much of the background variations as possible. For simplicity all variations were parametrized by hour angle. Trends were then removed through second order by fitting a second-degree polynomial to the individual integrations grouped in doublets of patch pairs, allowing a "jump" in the constant term of the polynomial between each of the two pairs (see fig. 2). If trend removal is successful, the constant term values are just the estimates of the D values for each of the patch pairs. We infer from trend removal experiments with simulated data, and a study of higher-order polynomial fits to the actual data, that this procedure in no way smooths the data in the sense of producing jump estimates biased towards smaller values. In processing the data, 30 obvious outliers were rejected from the entire set of about 10,000 integrations.

The actual calculations did not deal with the set of estimates of the $\{D_l\}$ directly (where l is an index over patch pairs) but a set, $\{U_m\}$, of independent sums of adjacent column differences between the D_l values, averaged over runs.

Values of U_m^2 for regions 100 and 400 are shown in table 3. Also shown are the corresponding variance components due to receiver noise, σ_m^2. Relative magnitudes of these variances reflect the quantity of data in each U_m estimate. Regions 200 and

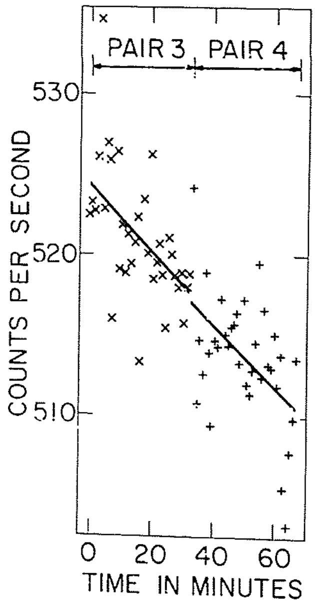

FIG. 2.—Best piecewise second-order polynomial fit to patch-pair doublet 3, 4 (run 2, region 100, tabulated in table 2). Only the constant term of the polynomial is independently adjusted for each patch-pair. The most obvious trend in the data is differential atmospheric emission decreasing to the right, with decreasing zenith angle.

TABLE 3

TABULATION OF U_m^2 AND σ_m^2 FOR SKY REGIONS 100 AND 400*

U_m^2	σ_m^2	U_m^2	σ_m^2
Region 100		Region 400	
0.376	0.672	1.296	0.516
0.004	0.315	0.000	0.561
0.001	0.166	0.572	0.318
0.129	0.092	0.653	0.191
0.010	0.058	0.076	0.135
0.259	1.556	0.106	1.136
0.746	4.452	0.644	2.629
0.007	2.548	0.308	9.151
0.531	0.423	0.753	7.711
0.052	0.097		

* See text for details.

300 have essentially no statistically significant bearing on our results. Together they contain about 20 percent of the data, and this figure is reduced to less than 10 percent after trend removal.

IV. UPPER BOUND FOR SKY ROUGHNESS

The Neyman-Pearson lemma conveniently prescribes the critical region for the most powerful test between two alternative hypotheses to be of the form

$$f_1/f_0 > \gamma ,$$

where f_1 and f_0 are the probability distributions of the samples under the two alternatives and γ prescribes the critical region (which may be used to define confidence level, as discussed by Lindgren 1968). If we wish to test the hypothesis that $\sigma_{\text{sky}}^2 = 0$ against $\sigma_{\text{sky}}^2 = \hat{\sigma}^2$ (where $\hat{\sigma}^2$ is some nonzero value), we have

$$\frac{\prod_m [2\pi(\sigma_m^2 + \hat{\sigma}^2)]^{-1/2} \exp [-U_m^2/2(\sigma_m^2 + \hat{\sigma}^2)]}{\prod_m [\sigma_m(2\pi)^{1/2}]^{-1} \exp (-U_m^2/2\sigma_m^2)} > \gamma .$$

The assumption that the $\{U_m\}$ are normally distributed is well justified by their composition through several averaging processes. Taking logs and simplifying, we obtain

$$s = \sum_m \frac{U_m^2}{\sigma_m^2(\sigma_m^2 + \hat{\sigma}^2)} > \gamma$$

as the statistic appropriate to the most powerful test. Since by definition the terms $U_m^2/(\sigma_m^2 + \hat{\sigma}^2)$ are independent unit-normal variables for $\hat{\sigma}^2 = \sigma_{\text{sky}}^2$, it follows that the statistic, s, would be a χ^2 variable except for the additional σ_m^2 factor in the denominator, which may be thought of as a weight. However, if we define

$$R = \left[\sum_m (\sigma_m^{-2})\right] \Big/ \left[\sum_m (\sigma_m^{-2})^2\right] = 0.109 ,$$

then the product Rs is approximately distributed as χ^2 on ν_{eff} degrees of freedom:

$$\nu_{\text{eff}} = R \sum_m 1/\sigma_m^2 = 8.03 .$$

The data then yield the conclusion that the hypothesis $\hat{\sigma}^2 \geq 0.055$ (counts per second)^2 can be rejected at the 90 percent level (i.e., at $R\gamma = 3.5$ for this χ^2 test).[3] That is, there exists only a 10 percent chance that a given random sample $\{U_m\}$ could exhibit as low a variance and still have σ_{sky}^2 as large as 0.055 (counts per second)^2. We therefore conclude with 90 percent confidence that σ_{sky}^2 is less than 0.055 (counts per second)^2, or

$$\sigma_{\text{sky}} < 4.3 \times 10^{-3}\ {}^\circ\text{K} \tag{3}$$

on an angular scale implied by an effective beam solid angle of 3.6 (arc min)^2.[4]

Separate consideration of regions 100 and 400 show no significant difference in roughness for those two parts of the sky. This is of interest since, as we have noted, region 400 lay in the plane of the Galaxy.

V. OBSERVATIONS OF M31

Before discussing conclusions to be drawn from our upper limit on σ_{sky}, we describe briefly a related series of measurements on M31. Our rationale was the following: If the microwave background is fairly isotropic on a small angular scale, it cannot be produced by "exotic" objects such as QSOs or Seyfert galaxies; we shall see that they are too few in number. Could it be, however, that all ordinary galaxies radiate in the microwave region, thereby producing in sum the microwave background? As a partial check on this idea, we attempted to measure the 3-mm flux from a nearby and familiar galaxy, M31. The standard "ON-OFF" technique was used for this set of measurements. Three regions of M31 were chosen for study: two near the optical center, and one at the position of a radio source found by the Ohio survey at 1415 MHz (OA 35.3, Kraus, Dixon, and Fisher 1966). The coordinates (1970 epoch) and the results of several hours of measurements on these three regions are shown in table 4. No statistically significant flux was observed from M31. We conservatively adopt the sum of the entries in table 4 plus two standard deviations as the upper limit to the antenna temperature of M31. This value is 0.005° K. Similar low limits have subsequently been set by Hobbs and Marionni (1971) on six other, more distant, spiral galaxies. Using our figure for M31 and equation (2), we find that 1.8×10^9 galaxies per steradian of comparable microwave luminosity would be necessary to produce an antenna temperature of 2.7° K. If we now assume that the sources contributing to the microwave background are on the average α times farther away than M31, then the number of sources of the same absolute microwave luminosity rises to $\alpha^2 \times (1.8 \times 10^9)$ per steradian.

TABLE 4

MEASUREMENTS ON M31

Area	α (1970)	δ (1970)	Antenna Temperature (° K)
1.	$0^h41^m04^s$	+41°06′30″.5	+0.0025 ± 0.0072
2.	0 41 06	+41 06 30″.5	−0.0020 ± 0.0076
3.	0 40 59	+41 08 49	−0.0113 ± 0.0072

NOTE.—The tabulated errors in the antenna temperature are standard deviations of the means.

[3] We have investigated this approximation by calculating the distribution of s using customary series representation methods. The result of hypothesis testing with this more precise distribution function was a slightly but not significantly smaller σ_{sky}.

[4] It should be stated as an indication of the power of this test that the hypothesis, $\sigma_{\text{sky}}^2 = 0$, would be correctly diagnosed with a probability of 0.7 for the given critical region.

VI. DISCUSSION

The main result of this experiment ($\sigma_{sky} < 0.0043°$ K) may be analyzed in two ways. First, it sets an upper limit on the small-scale anisotropies produced in a "smooth" background of primeval fireball radiation by density inhomogeneities, gravity waves, etc. Alternatively, the result may be used to set limits on the parameters for discrete-source models for the microwave background radiation.

a) Fireball Model

Our value of σ_{sky} is in terms of antenna temperature. Thus to find the corresponding value for the fractional intensity fluctuation, $\Delta I/I$, we must divide σ_{sky} by the antenna temperature of the microwave background at $\lambda = 3.5$ mm. In the fireball picture, where the spectrum is assumed to be blackbody, the antenna temperature at $\lambda = 3.5$ mm falls well below the thermodynamic temperature of 2.7° K (since we are no longer in the Rayleigh-Jeans region). It is in fact 1.16° K, so we have

$$\Delta I/I \leq 3.7 \times 10^{-3}$$

as the upper limit on fluctuations on an angular scale ~80″ in the fireball radiation.

The measurement is not sufficiently sensitive to fit any of the models of Sunyaev and Zel'dovich (1970) or Longair and Sunyaev (1970). It does, however, lower the allowable upper limit on the energy density in the form of gravity waves of wavelength less than or equal to 1 Mpc (Dautcourt 1969).

We emphasize that the results of Parijskij and Pyatunina (1970) at $\lambda = 4$ cm may be used to set even more stringent limits on possible small-scale anisotropies in the fireball picture. They estimate $\sigma_{sky} \leq 0.0007°$ K (with no confidence level given). Recall that the antenna temperature of a 2.7° K blackbody spectrum at 4 cm is exactly 2.7°. Thus

$$\Delta I/I = \sigma_{sky}/T \leq 2.6 \times 10^{-4} \tag{4}$$

for their experiment, with a 1′.4 × 20′ beam.

b) Discrete-Source Models

In such models the microwave background radiation does not have a blackbody spectrum. For instance, in the models of Wolfe and Burbidge (1969), the spectrum at 3.5 mm is well above a 2.7° blackbody curve. In discussing discrete-source models, we will make the simplest assumption about the spectrum, namely, that it is graybody, as suggested by Shivanandan, Houck, and Harwit (1968), and that it fits the long-wavelength radiometric measurements of Stokes, Partridge, and Wilkinson (1967). Then the antenna temperature at 3.5 mm, as at longer wavelengths, is 2.7° and

$$\Delta I/I \leq 1.6 \times 10^{-3} . \tag{5}$$

The implications of this upper limit for discrete-source models may be seen in two ways. One, the method of Parijskij and Pyatunina, is to calculate the minimum number of sources (assumed equally luminous) per steradian which are required to produce the observed uniformity of the background. Using our experimental values (2) and (5), we find a surface density $N_S \geq 13 \times 10^{11}$ sterad^{-1}.

Alternatively, we may fit our result into the framework of Smith and Partridge (1970). The *minimum* value for their parameter $\mu = N\Omega(\Delta I/I)^2$ for any model was 3×10^{-6}, where N is the present number density of sources per Mpc^3. Inserting our values for Ω and $(\Delta I/I)$, and solving for the number density of sources, we find

$$N \geq 0.33 \text{ Mpc}^{-3} , \tag{6}$$

about equal to a commonly accepted value for the number density of galaxies ($0.46 \times H_0/100$ Mpc^{-3}, where H_0 = Hubble constant in km s^{-1} Mpc^{-1} [Kiang 1961]). In other words, a majority of galaxies must be microwave sources if the sum of the radiation from all sources is to be "smooth" enough to agree with the observations.[5]

Our result (6) rules out *a fortiori* models in which a limited class of objects, say QSOs or Seyferts, produce the radiation. Likewise, the result rules out models such as Wagoner's (1969) where the radiation is emitted predominantly in a single spectral line. In such a model, the radiation we receive at a fixed observing wavelength comes only from a limited number of sources with a narrow range of redshifts. On the other hand, our result cannot rule out models having more sources than there are galaxies.

Now let us reconsider in more detail the possibility that ordinary galaxies emit microwave radiation and add together to produce the microwave background. The result (6) already casts doubt on this suggestion, since it appears that one needs a very large number of such sources. This conclusion is reinforced when we consider the results of our measurements on M31. The sources contributing to the radiation are likely to be at least 100 times farther away on the average than M31. Such a distance, 60 Mpc, corresponds to a redshift of only 0.02. If we take $\alpha = 100$, the surface density of sources becomes $N = 1.8 \times 10^{13}$ sterad^{-1}, and N rises to about 4.5 Mpc^{-3}, 10 times the number density of galaxies (Kiang 1961).

We would like to thank the staff of Kitt Peak National Observatory for their hospitality during our stay there, and especially the night operators of the 36-foot telescope who spent long hours measuring apparently blank sky. Conversations with E. K. Conklin and N. J. Woolf at Kitt Peak and J. Deeter of the University of Washington were very helpful. Mr. Deeter was also instrumental in the computer analysis of our data.

REFERENCES

Allen, C. W. 1963, *Astrophysical Quantities* (London: Athlone Press).
Chibisov, G. V., and Ozernoy, L. M. 1969, *Ap. Letters*, **3**, 189.
Conklin, E. K., and Bracewell, R. N. 1967, *Nature*, **216**, 777.
Dautcourt, G. 1969, *M.N.R.A.S.*, **144**, 255.
Dicke, R. H., Peebles, P. J. E., Roll, P. G., and Wilkinson, D. T. 1965, *Ap. J.*, **142**, 414.
Epstein, E. E. 1967, *Ap. J. (Letters)*, **148**, L157.
Hobbs, R. W., and Marionni, P. 1971, *Ap. J.*, **167**, 85.
Kiang, T. 1961, *M.N.R.A.S.*, **122**, 263.
Kraus, J. D., Dixon, R. S., and Fisher, R. O. 1966, *Ap. J.*, **144**, 559.
Lindgren, B. W. 1968, *Statistical Theory* (2d ed.; New York: Macmillan Company).
Longair, M. S., and Sunyaev, R. A. 1969, *Nature*, **223**, 719.
Parijskij, Yu. N., and Pyatunina, T. B. 1970, *Astr. Zh.*, **47**, 1337 (English transl., *Soviet Astr.—AJ*, **14**, 1067).
Peebles, P. J. E., and Wilkinson, D. T. 1968, *Phys. Rev.*, **174**, 2168.
Penzias, A. A., Schraml, J., and Wilson, R. W. 1969, *Ap. J. (Letters)*, **157**, L49.
Rasband, S. N. 1971, *Ap. J.*, **170**, 1.
Rees, M. J., and Sciama, D. W. 1968, *Nature*, **217**, 511.
Salpeter, E. E., and Hazard, C. 1969, *Ap. J. (Letters)*, **157**, L87.

[5] At first glance, it may appear that the 4-cm observations of Parijskij and Pyatunina (1970) provide a more stringent limit on N in the discrete-source model. However, one must recall that there is a special advantage of working at short wavelengths, already referred to in our Introduction. Consider the models put forward by Wolfe and Burbidge (1969; see also their note added in proof). If we require $\lambda_c > 1$ mm, we find from their table 4 that observations at 3.5 mm should exhibit a $(\Delta I/I)^2$ more than 5 times larger than observations at 3.95 cm for all the models Wolfe and Burbidge consider. This is simply because their parameter $N_\omega(z_{1/2})$ decreases sharply as the wavelength of observation decreases toward λ_c. Also, if one uses the Gaussian beam-profile approximation mentioned earlier, the value for Ω describing our beam is about 18 times smaller than that of Parijskij and Pyatunina. Then, finally, assume that their limit is as conservative as our result. By combining all these factors, we find that our experiment is 3 times more sensitive a test for the discrete-source models of Wolfe and Burbidge.

Shivanandan, K., Houck, J. R., and Harwit, M. O. 1968, *Phys. Rev. Letters*, **20**, 1460.
Smith, M. G., and Partridge, R. B. 1970, *Ap. J.*, **159**, 737.
Stokes, R. A., Partridge, R. B., and Wilkinson, D. T. 1967, *Phys. Rev. Letters*, **19**, 1199.
Sunyaev, R. A., and Zel'dovich, Ya. B. 1970, *Ap. and Space Sci.*, **7**, 3.
Thorne, K. S. 1967, *Ap. J.*, **148**, 51.
Wagoner, R. V. 1969, *Nature*, **224**, 481.
Wolfe, A. M., and Burbidge, G. R. 1969, *Ap. J.*, **156**, 345.