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Strange attractors: new tools for describing chaos

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L'Ordre dans le Choas and Deterministic Chaos: An Introduction

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Strange attractors: New tools for describing chaos

L'Ordre dans le Chaos

P. Bergé, Y. Pomeau, Ch. Vidal
353 pp. Hermann, Paris, 1984. 230 FF

Deterministic Chaos: An Introduction

H. G. Schuster
220 pp. Physik-Verlag, Weinheim (US dist. Verlag Chemie, Deerfield Beach, Florida), 1984. $43.75

Reviewed by Jerry P. Gollub

A substantial change has taken place recently in the way scientists view unpredictable or apparently random phenomena. In addition to the lack of predictability that comes either from large numbers of particles or from quantum effects, it is now clear that even macroscopic systems having only a few degrees of freedom can be unpredictable. Examples are hydrodynamic flows near the onset of turbulence, time-dependent chemically reacting systems, laser amplifiers, mechanical oscillators, acoustic waves, plasmas and many other systems. The unifying feature that characterizes all of these phenomena is that the governing macroscopic equations are strongly nonlinear. In each case, parameters can be found for which trajectories describing the evolution of the system in phase space exhibit a sensitive dependence on initial conditions, so that nearby trajectories diverge from each other exponentially in time. If the system is dissipative and driven, the trajectories wander erratically on a "strange attractor," a set that is topologically complicated. In the simplest case, that of a three-dimensional phase space, the attractor is composed of an infinite number of thin sheets having zero total volume but unbounded area. A strange attractor is thus an example of a fractal, an object of fractional Hausdorff dimension. Systems whose (deterministic) dynamics are described by a strange attractor are generally called chaotic, and may be contrasted with stochastic systems, where noise terms appear in the equations of motion.

From a mathematical point of view, most of what is known about chaos and about the bifurcations leading to chaos is based on analyzing nonlinear mappings rather than on the partial differential equations that are of physical interest. Such mappings often contain the essential features (even quantitative ones) of the transition to chaos. Both of the books under review emphasize nonlinear mappings and their relationship to recent experiments.

In my opinion, chaotic phenomena deserve a place in the curriculum of classical mechanics because they enrich our understanding of the behavior of even the simplest mechanical systems, such as the pendulum. However, most of the books and reviews concerned with chaotic phenomena are too technical to be pedagogically useful, or even to instruct most physicists not already familiar with the subject.

L'Ordre dans le Chaos (English translation to be published by Wiley, New York, 1986) takes an important step toward making the basic ideas about chaos and related nonlinear phenomena easily accessible both to interested novices and to students who have learned undergraduate mechanics. The authors have made major contributions to the subject, especially to the connection between chaos and hydrodynamic phenomena. Their discussions of the basic topological ideas (sensitive dependence on initial conditions, stretching and folding of orbits and fractal dimension) are lucid. The book gives elementary examples of strange attractors, including some that come from numerical simulation and others based on experimental data. The major routes to chaos (involving quasiperiodicity, subharmonic bifurcations and intermittency) are treated in depth. The book is also strong in its discussion of methodological issues: bifurcation theory, the use of Poincaré sections and techniques for the characterization of experimental data. Many of the mathematical ideas needed for an understanding of chaos are discussed in the book, but not in great depth.

The mathematical aspects of chaos are subtle and still the subject of active research. Those interested in a proper mathematical description may find this work unsatisfying and will have to consult a more technical book such as *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, by J. Gluckenheimer and P. Holmes (Springer-Verlag, New York, 1983; for a review see PHYSICS TODAY, November 1985, page 102). A discussion of conservative (Hamiltonian) systems is also missing from the present work. However, those interested in learning about chaos for the first time, or in supplementing a mechanics course, will certainly find *L'Ordre dans le Chaos* far more accessible.

Schuster's book is a graduate-level research monograph. It contains an extensive mathematical discussion of chaotic mappings and the routes to chaos, including the origin of self-similarity in the structure of the strange attractors and the universal exponents that describe the approach to chaos by period doubling. Schuster also discusses irregular motion in conservative systems and the question of whether quantum chaos exists. The list of references is extensive and quite useful, but has some prominent omissions.

Deterministic Chaos contains significant inaccuracies: It presents the seminal work on the onset of turbulence by David Ruelle and Floris Takens imprecisely, describes the power spectrum of chaotic behavior incorrectly as being broad only at low frequencies and gives the impression that the driven pendulum is always chaotic above a critical driving amplitude. I felt the author's evaluation of experiments to be uncritical and sometimes misleading. For example, in the discussion of chaos in thermal convection he neglects to point out the importance of the size of the fluid layer. (Large layers do not follow the simple routes to chaos.) Finally, Schuster often assumes the reader already knows much about critical phenomena and is inter-

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ested in the analogy between chaotic transitions and phase transitions.

Despite these limitations, Schuster's book should be useful to some readers because so much of the primary literature on chaos is obscure. However, *L'Ordre dans le Chaos* should be more successful in communicating some of the novel ideas about nonlinear physics to interested nonspecialists and students.

Kapitza, Rutherford and the Kremlin

Lawrence Badash

129 pp. Yale U.P., New Haven, 1985. $20.00

One of the main obstacles to doing research in the history of Soviet science is the lack of useful primary sources that provide information outside the framework of the Communist Party line. *Kapitza, Rutherford and the Kremlin*, by Lawrence Badash, is of great importance because it draws from several dozen semiprivate letters of a major Soviet scientist that were made public only recently.

These letters were written in English by Peter Kapitza to his wife Anna in 1934–35, at a turning point in his career. Kapitza was then in the USSR, while Anna lived in Cambridge (Great Britain) with their two children. The circumstances that temporarily separated this family and forced them to correspond in English were most unusual: Between 1921 and 1934 Kapitza was allowed by the Soviet government to live and work in the West, doing research in the Cavendish Laboratory under Ernest Rutherford, while retaining his Soviet citizenship and making almost annual trips to the USSR. He became a leading authority in low-temperature physics and magnetism by 1934. He was elected a Fellow of the Royal Society, and a special research institution, the Mond Laboratory, was created for him in Cambridge. At this high point in his career, his own government suddenly, in the summer of 1934, prevented his return to Cambridge during one of his visits to the USSR.

The letters in question were written between November 1934 and July 1935. Writing in English allowed Kapitza to limit the number of possible Soviet readers and to make the letters also accessible, if need be, to his British colleagues, Rutherford in particular. The book also includes two letters from Kapitza to Rutherford, written in May 1935 and March 1936, as well as Kapitza's letter of May 1935 to Viacheslav Molotov, prime minister of the Soviet Union at the time.

Although the letters are not reprinted in their complete form (they were excerpted by Kapitza's wife) and do not cover the period immediately after Kapitza's detention in the late summer of 1934, they still create a complex and revealing picture.

As a result of Badash's research, we can conclude that Kapitza was able to overcome the shock of his detention and continue his career in the harsh social environment of Stalinist Russia because he understood quickly that he could not rely on his Soviet colleagues for help or support. They were either unwilling or powerless. In his letters Kapitza complains bitterly about the lack of support from his Soviet colleagues. He singles out his teacher Abram Ioffe as being particularly distant and cold.

About 15 years ago I had an opportunity to learn something about Ioffe's motives from Ioffe's closest assistant, Yakov Dorfman. Dorfman was a prominent Soviet physicist and historian of science; in the 1920s and 1930s he was one of the leading authorities on magnetism and, in a way, Kapitza's rival. Dorfman, as Ioffe's assistant, was involved in the planning of Kapitza's visit to the USSR in 1926—a visit that never materialized. Badash mentions that in 1926 Kapitza received a formal request from the Soviet Government, signed personally by Leon Trotsky, to come to the USSR for consultation. Dorfman claims to have personally procured Trotsky's signature under the direct order of Ioffe. Trotsky readily signed the letter, and Ioffe hoped that Kapitza would react with due respect to this invitation. Kapitza's response came on a piece of paper torn out from a notebook, with four short words in longhand: "Cannot come: too busy." According to Dorfman, this note enraged Ioffe. He feared such behavior toward the Soviet government would ultimately harm not only Kapitza and Ioffe himself, who had initiated Kapitza's extended stay in Great Britain, but also the whole Soviet physics community.

Ioffe was so concerned that he entrusted Dorfman with a special assignment: During Dorfman's forthcoming trip to Europe he was to drop by Cambridge, have a tête-à-tête with Kapitza and inform him of Ioffe's frustration.

Dorfman indeed met Kapitza in England and expressed Ioffe's concern with Kapitza's behavior. Kapitza responded: "It is not a matter of politesse. You should understand, Yakov, that we British scientists are very proud and independent people." Dorfman responded: "If you consider yourself a British scientist, then my remarks are inappropriate. I should apologize and leave." Of course Kapitza had to beat a retreat. He apologized, saying that it was just a joke, that of course he considered himself a Soviet scientist and would act accordingly.

This must have been an awkward moment for a proud and independent man. It seems he never forgot it, and never forgave Yakov Dorfman, the bearer of bad news. Years later, Dorfman believed, Kapitza played a prominent role in blocking his election to the Soviet Academy of Sciences. When Dorfman died in 1974, his funeral was attended by two of the most accomplished Soviet physicists, Vitaly Ginz-

Outside the Cavendish. This photo, taken by C. E. Wynn-Williams on 4 June 1929, shows P. M. S. Blackett, Peter Kapitza, Paul Langevin, Ernest Rutherford and C. T. R. Wilson in front of the Cavendish Laboratory. The photo appears in the book under review.