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Ian S. Blecher  
**Kant on Formal Modality**

**Abstract:** I propose to explain Kant’s novel claim, in the *Critique of Pure Reason*, that all judgments have a formal modality. I begin by distinguishing the modality of a judgment’s form from the modality of its content, and I suggest that the former is peculiar in merely affecting the subject’s understanding of his own act of judging. I then contrast the modal account of such an understanding (in terms of the possibility and actuality of a judgment) with the traditional, non-modal understanding of it (in terms of the giving and withholding of assent). I conclude by suggesting that Kant prefers the former because he conceives of knowledge on Aristotle’s model: as a progress in the mind from capacity to act.

**Keywords:** Logic, theory of judgment, modal theory, Aristotle.

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*Ohne modalität ist gar kein Urteil möglich ...  
Refl 3111 (AA 16: 663, 1770s)*

I

1. It is a distinctive feature of the system of logic that Kant presents in the *Critique of Pure Reason* that judgments,<sup>1</sup> as such, have a modality – and in two respects.

With respect to their *contents* – i.e. the specific representations they comprise – judgments include the concept of an object as possible or impossible; existent or non-existent; or necessary or contingent. Kant calls these concepts the “categories of modality” (*Kategorien der Modalität*); I will consider them in another article. In this article I am only going to consider the modality of judgments

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<sup>1</sup> By “judgments” I mean what Kant calls, in the *Critique of Judgment*, “logical” judgments (KU § 1, AA 05: 203ff.). “Aesthetic” judgments – for example, judgments concerning the beautiful – are constituted by a feeling, and so cannot be knowledge. They therefore require separate treatment. (That said, Kant does ascribe to aesthetic judgments, as such, a modality [KU §§ 18–22, AA 05: 236–240].)

with respect to their *form* – that is, their logical form, the manner in which contents are ordered in them.<sup>2</sup>

Formal modality is a determination of what Kant calls “affirmations” and “negations”, and what I will call, more generally, “predications”. A predication is a logically articulate relation of concepts. This can stand on its own (as in “All men are mortal”), or in relation to another predication (“If all men are mortal, then Caius is mortal”, “Either all men are mortal, or all men are non-mortal”).<sup>3</sup> A formal modality consists in the representation of the “value” of a predication as such (or as he puts it, its “value in relation to thinking in general”<sup>4</sup> [KrV, A 74/B 100]). It seems to be Kant’s view that this value is what distinguishes judgments from *mere* predications: for does not just relate concepts, but also assigns them a certain cognitive significance.

He will eventually distinguish three formal modalities, corresponding to the three pairs of modal categories. A judgment is “problematic” (*problematisch*) if it represents a predication as “merely possible” (or “arbitrary”) (*bloß möglich [beliebig]*); it is “assertoric” (*assertorisch*) if it represents the predication as “actual” (or “true”) (*wirklich [wahr]*);<sup>5</sup> and it is “apodeictic” (*apodiktisch*) if it represents the predication as “necessary” (*notwendig*) (KrV, A 74–75/B 100–101).<sup>6</sup>

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2 The distinction between material and formal modality has sometimes been interpreted as a distinction between modality *de re* and modality *de dicto*. (Cf. Wolff, Michael: *Die Vollständigkeit der kantischen Urteilstafel*. Frankfurt am Main 1995, 126; Dicker, Georges: *Kant’s Theory of Knowledge: an Analytical Introduction*. New York 2004, 57). It is worth noting, however, that nothing in the *Critique of Pure Reason* requires that interpretation, and in fact it is inept to the extent that modality *de dicto* can be construed as a determination of content. (I will say a little more about this below.) A closer comparison with formal modality would be Frege’s concept of “sentential force”. But this also is inexact, since for Frege force does not necessarily have modal significance.

3 Today such relations are commonly called “propositions”. I will speak of “predications”, however, because the term “propositions” is liable to cause confusion with Kant’s term “Sätze”. His will speak of “Sätze” in various ways (cf. Patzig, Günther: “Die logischen Formen praktischer Sätze in Kants Ethik”. In: *Kant: zur Deutung seiner Theorie von Erkennen und Handeln*. Ed. by Gerold Prauss. Köln 1973) – but when he is being careful, the term designates assertoric judgments rather than mere relations of concepts (JL § 30: n2, AA 09: 109; cf. R 3111, AA 16: 663; V-Lo/Wiener, AA 24: 934).

4 “Wert der Copula in Beziehung auf das Denken überhaupt”.

5 Note that an assertoric judgment, for Kant, is not an “assertion” in the contemporary sense of “say[ing] something in such a manner as deliberately to convey the impression of saying it with the overriding intention of saying something true,” as Dummett has defined it. (Dummett, Michael: *Frege: Philosophy of Language*. Cambridge, MA, 1981, 300.) Nor is it this act *in foro interno*. It is simply the representation of a predication as true.

6 In the *Logic* manuscript prepared by G.B. Jäsche, Kant compares these values to those associated with “opinion” [“*Meinen*”], “belief” [“*Glauben*”], and “science” [“*Wissen*”] respectively (Log

Since Kant's view is that all judgments, with respect to logical form, are modally determinate, the proposition I am going to consider is that

(M) *All judgments are as such problematic, assertoric, or apodeictic.*

(Although Kant's description of the formal modalities suggests that he takes this disjunction in an exclusive sense – at least with regard to problematic and assertoric judgments – I will consider an alternative interpretation below.)

2. I said that (M) is distinctive in the history of logic. In fact it is distinctive in two ways. It is distinctive in its terminology: though they have obvious cognates in the tradition,<sup>7</sup> the terms “problematic”, “assertoric”, and “apodeictic” are Kant's invention.<sup>8</sup> And, as this itself suggests, (M) is also distinctive in its conception of judgment.

Perhaps the easiest way to bring this out is to consider a traditional conception of modality – that of Georg Friedrich Meier's *Auszug aus der Vernunftlehre*:

The representation of the manner in which the predicate does or does not belong to the subject is the determination of the concept of their combination and its negation (*modus formalis*). Either a judgment has such a determination or it does not. The former is an impure judgment (*iudicium modale, modificatum, complexum qua copulam*), e.g. ‘This world necessarily exists’, ‘It does not necessarily exist’. The latter however is a pure judgment (*iudicium purum*).<sup>9</sup>

The most obvious sense in which Meier's conception differs from Kant's is in distinguishing a “non-modal” class of judgments – apparently, those from which the concepts of necessity, contingency, possibility and impossibility are omitted. (To add such a concept, then, will literally be to modify the judgment.)

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§ IX, AA 09: 65 ff.). The comparison can be instructive, but it is important to remember that while the latter have primarily a *subjective* significance, the former have primarily a logical one: the fact that, for example, problematic judgments express opinions is not part of the account of them as judgments.

<sup>7</sup> Aristotle, for example, speaks in the *Topics* of “*problemata*” (101b29), i.e. what is to be vindicated or refuted in a dialectical argument; and, in the *Posterior Analytics*, of “*apodeixeis*” (A.2, 71b9 ff.), i.e. what is demonstrated by means of a proper syllogism.

<sup>8</sup> See Tonelli, Giorgio: “Die Voraussetzungen zur Kantischen Urteilstafel in der Logik des 18. Jahrhunderts”. In: *Kritik und Metaphysik. Heinz Heimsoeth zum achtzigsten Geburtstag*. Ed. by F. Kaulbach and J. Ritter. Berlin 1966, 156.

<sup>9</sup> Halle, 1752, § 309: “Die Vorstellung der Art und Weise, wie das Prädicat dem Subjekte zu oder nicht zukommt, ist *die Bestimmung des Verbindungsbegriffs und der Verneinung desselben* (modus formalis). Ein Urteil hat entweder ein solche Bestimmung, oder nicht. Jenes ist *ein unreines* (iudicium modale, modificatum, complexum qua copulam), dieses aber *ein reines Urtheil* (iudicium purum), z. E. diese Welt ist notwendig da, sie ist nicht nothwendig da.”

There is also, however, a more fundamental difference. Kant points to this himself, in a lecture – evidently on Meier<sup>10</sup> – from about 1780. Logicians in the tradition, he says,

did not take the division of that which concerns the modality of judgments so precisely as we do, but called every concept of combination ‘modality’. [Take] for example [the judgment], ‘the world exists in a necessary manner’. Here the word in a *necessary manner* was the modality. But can logic really judge whether a thing is necessary or not? No, for it has nothing to do with things and their necessity. Accordingly it can ask only whether a judgment is *expressed* with necessity or not. [V-Lo/Wiener, AA 24: 935, emphasis added.]<sup>11</sup>

In denying that the representation of necessary existence belongs to logic, Kant is not denying that it is a modality in any sense.<sup>12</sup> He is denying that it is a *logical* modality – that it belongs to the logical form of judgments (how they are “expressed”). For, he thinks, the representation of the manner in which a thing exists pertains to the content of judgment, and not the mere form of representation.

By itself, this would not be noteworthy. Apparently Meier himself had denied that representations of possible and necessary existence belong to logic in the strictest sense, for he had classified the judgments which contain them as “impure”.<sup>13</sup> Indeed this was the predominant view among logicians in Kant’s time. According to J.H. Lambert, for example, modal concepts “belong to ontology, and do not depend merely on the external [sc. logical] form of knowledge”. And according to Baumgarten, in a commentary on Wolff’s *Logica*, “modes [sc. modalities] are treated contingently in logic” – so that logicians

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<sup>10</sup> The *Auszug aus der Vernunftlehre* had formed the basis of Kant’s standard course of lectures on logic since the 1750s (see AA 01: 503). He therefore knew it particularly well; his annotations on it would form the basis for the 16th volume of the “Academy” edition of the works of Kant.

<sup>11</sup> “Was die Modalitaet der Urtheile betrifft, so haben die Alten die Eintheilung nicht so genau genommen als wir, sondern jeden Verbindungsbegriff nannten sie Modalitaet. Z.B. die Welt ist nothwendiger Weise da. Hier war bei ihnen das Wort: nothwendiger Weise, die modalitaet. Aber kann die Logic wohl urtheilen, ob ein Ding nothwendig sey oder nicht? Nein, denn sie hat nichts mit den Dingen und ihrer Nothwendigkeit zu tun. Daher kann sie nur fragen, ob ein Urtheil mit einer Nothwendigkeit ausgedrückt sey, oder nicht?”

<sup>12</sup> Sellars seems to read him that way. See Sellars, Wilfrid: *Science and Metaphysics: Variations on Kantian Themes*. New York 1968, 54f.

<sup>13</sup> Meier never defines “purity” and “impurity” in the *Auszug aus der Vernunftlehre*. But the glosses he offers – e.g. “*modificatum*” – suggest he thinks of these concepts as derivative classifications of judgment.

may omit them if they like.<sup>14</sup> Indeed, Wolff had omitted them.<sup>15</sup> Nor was he alone in this.<sup>16</sup>

What is distinctive in Kant's logic is the positive characterization of the logical form of judgments as involving modality. Traditionally, logicians had characterized judging, as opposed to mere predicating, in terms of what they called "assent" or sometimes "positing" (Latin: *assensus*, *ponere aliquid*; German: *annehmen*).<sup>17</sup> This characterization shares something with Kant's: to assent to a predication is to represent it as true rather than arbitrary (see Refl 2506, AA 16: 397, where an analogy is explicitly drawn.) But there is also an important difference. Assent was never characterized as a *modality*. It did not represent a predication as actual; it was not opposed to the representation of the same predication as merely possible. It was opposed to the withholding of assent – as in an hypothesis or an expression of doubt.

There is nothing obviously wrong with the traditional characterization. Kant himself had accepted it for much of his career; it is, for example, in the "Blom-

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**14** Baumgarten, Alexander Gottlieb: *Acroasis Logica in Christianum L. B. de Wolff*. Halle 1761, § 160; Lambert, Johann Heinrich: *Neues Organon oder Gedanken über die Erforschung des Wahren und der Unterscheidung von Irrtum und Schein*. Leipzig 1764, Book i, § 137. Cf. Reusch, Johann Peter: *Systema logicum antiquorum atque receptorum item propria praecepta exhibens*. Jena 1734, §§ 376ff.; Gesner, Johann Matthias: *Primae lineae isagoges in eruditionem universalem*. Leipzig 1756, Vol. 2, §§ 1050f.; Baumeister, Friedrich Christian: *Institutiones Philosophia Rationalis methodo Wolfii conscriptae*. Wittenberg 1736, § 201. and Meier, *Auszug aus der Vernunftlehre*, § 168. This is not to say a distinction between judgments and predications was rigorously maintained; logic textbooks often referred to what I am calling "predications" as "judgments". Nonetheless, it was maintained, contrary to what is sometimes thought (see Geach, P. T.: "Assertion". In: *Philosophical Review* 74, 1965, 449–465). For a discussion of the point, see Owen, David: "Locke on Judgment". In: *The Cambridge Companion to Locke's Essay*. Ed. by Lex Newman. Cambridge 2007, 412ff.).

**15** They are of course discussed at length in his *Ontologia* (Frankfurt am Main and Leipzig 1736), as Longuenesse notes. (See Longuenesse, Béatrice: *Kant and the Capacity to Judge: Sensibility and Discursivity in the Transcendental Analytic of the "Critique of Pure Reason"*. Princeton 1998, 158n.)

**16** Adickes notes that H. S. Reimarus, for example, had omitted modalities from his *Vernunftlehre*. See Adickes, Erich: *Kants Systematik als systembildender Factor*. Berlin 1987, 38.

**17** See Aristotle: *De Interpretatione*. Oxford 1936, 17a25ff.; Locke, John: *An Essay Concerning Human Understanding*. New York 1975, Book IV, Chap. XIV, § 3 *et passim*; Hume, David: *A Treatise of Human Nature*, bk. I: pt. III, § 7; Wolff, *Logica*. New York 1978, § 1009; and Meier, *Auszug aus der Vernunftlehre*, § 168. This is not to say a distinction between judgments and predications was rigorously maintained; logic textbooks often referred to what I am calling "predications" as "judgments". Nonetheless, it was maintained (contrast Geach, P. T.: "Assertion". In: *Philosophical Review* 74, 1965, 449–465). For a discussion of the point, see Owen, David: "Locke on Judgment". In: *The Cambridge Companion to Locke's Essay*. Ed. by Lex Newman. Cambridge 2007, 412ff.).

berg” transcript of his logic lectures from 1770 (AA 24: 277) and the “Philippi” transcript of lectures from the following year (cf. AA 24: 463). There is some evidence that his thinking began to change during the middle 1770s, but this consists mainly in handwritten *Reflexionen* (e.g. Refl 3111<sup>18</sup>), which are difficult to interpret. It is not until the “Vienna” lectures of 1780 that he begins to speak of “modality” in the logico-formal sense; and it is only in 1781, with the first edition of the *Critique of Pure Reason*, that he introduces the terms “problematic”, “assertoric” and “apodeictic” to designate specific modalities.<sup>19</sup>

It is not clear what led to this change. I am not aware of any work in which he discusses it.<sup>20</sup> A broad motivation, however, is suggested by the chronology: Kant was developing his distinctive conception of judgments as involving logical modality at the same time he was developing the “critical” conception of judgment in general.<sup>21</sup> Particularly in view of the extraordinarily systematic character of that conception,<sup>22</sup> it seems likely that it is connected with the distinctive conception of modality that accompanies it.

To bring out the connection would require a whole book. Here I will only make a start, by considering, in a preliminary way, the “distinctive conception of modality” itself.

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**18** According to Adickes, *Reflexion* 3111 – a marginale criticizing Meier’s conception of modality – most likely belongs to Phase “ $\tau$ ” or “ $\nu$ ”, thus, ca. 1775–78. A slightly broader range of dates is also possible, but none earlier than 1772.

**19** He had already begun to use the terms “problematic” and “assertoric” in the “Blomberg” lectures: “All judgments,” he says (in emphatic Latin) “are either problematic or assertoric”, where the former are defined as “undetermined” with respect to the relation of concepts, and the latter as “positing” something in this respect (§ 301, AA 24: 276). It is apparent that these are not being construed as formal modalities.

**20** This is not surprising. In general, Kant’s later writings contain little assessment of his earlier views. A well-known exception is the introduction to the *Prolegomena*, where he speaks of the “dogmatic slumber” from which Hume awakened him (Prol, AA 04: 260). But even there he does not elaborate. (What “dogma” had he accepted? What in Hume had awakened him? He never says.)

**21** The famous letter to Marcus Herz, in which Kant first announces his plan for a criticism of reason, is dated 21 February, 1772 (Br 70, AA 10: 128–135).

**22** See, e.g. KrV, A 474/B 502: “Human reason is by nature architectonic, i.e it considers all knowledge as belonging to a possible system and therefore countenances only those principles which at least do not make it impossible for any envisaged knowledge to stand together in a system with the rest.” [Die menschliche Vernunft ist ihrer Natur nach architektonisch, d.i. sie betrachtet alle Erkenntnisse als gehörig zu einem möglichen System, und verstattet daher auch nur solche Principien, die eine vorhabende Erkenntniß wenigstens nicht unfähig machen, in irgend einem System mit anderen zusammen zu stehen.]

## II

1. A natural place to begin is section nine of the *Critique*,<sup>23</sup> where the formal modalities are introduced.

The purpose of that section is to give a systematic description of the logical form of understanding or judgment.<sup>24</sup> Following Boehm, perhaps,<sup>25</sup> this is accomplished by means of a table comprising twelve “moments” (or specific forms<sup>26</sup>) of judgments, three each under four headings: “quantity”, “quality”, “relation” and “modality”. The Table appears in both editions without a title; for convenience I will refer to it as the “Table of Logical Forms”.<sup>27</sup>

It is accompanied by four short appendices. These initially seem to consist in small correctives to traditional logical doctrine, for example, that singular judgments – e.g. “Caius is mortal” – are formally different from universal ones – “Every man is mortal”; and that affirmative judgments – e.g. “Caius is mortal” – are formally different from “infinite” ones – “The soul is non-mortal”;<sup>28</sup> and Kant probably does intend them that way. But he intends something more too.

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**23** Numbered paragraphs only appeared in the second edition (originally published in 1787). The text is more or less the same in both editions, however, and almost all I have to say holds as much for one as for the other.

**24** For Kant the faculty of understanding and the faculty of judgment are the same. “To understand”, for him, means to know by judging. See, e.g. KrV, A 69/B 94, A 81/B 106, A 97.

**25** See Boehm, Andreas: *Logica in Usum Auditorii sui Ordine Scientifico*. Frankfurt am Main 1749, § 104.

**26** Because all judgments originate in understanding, they share a generic form. This might be called the form of judgments. It is natural, however, to speak of the moments in which the form of judgments consists as themselves forms – for example, the “assertoric form of judgments”. This way of speaking will not cause confusion so long as it is not taken to imply an irreducible plurality of such forms.

**27** It is called the “Logical Table of Judgments” in the *Prolegomena* (§ 21, AA 04: 302). Perhaps for this reason commentators have tended to call it “the Table of Judgments”. That title, however, is imprecise, since judgments have both form and content. Probably it ought to be called the “Table of Moments of the Logical Function of Understanding in Judgments” (cf. KrV, A 69/B 94), but this, I take it, would be *too* precise.

**28** Infinite judgments can also be distinguished from merely negative ones. The former express an affirmative predication of a negated predicate, e.g. that non-mortality belongs to God. By contrast, the latter (e.g. “God is not mortal”) express a negative predication, i.e., that mortality does *not* belong to God. It is logically possible, then, for negative judgments to be infinite, e.g. “Caius is not non-mortal”. Perhaps the reason Kant does not draw attention to this contrast is that it can be drawn in terms of the relations between judgments, while the contrast between affirmative and infinite judgments can only be drawn by appealing to the relation of judgments to objects.



Traditionally, Kant says, a judgment's logical form was the manner in which it entered into the commonly accepted forms of inference – e.g. “Barbara” or “Darii”. Judgments that entered into the same forms of inference in the same manner were assigned the same logical form; the totality of logical forms was given by the totality of inference forms.<sup>29</sup> Although Kant regards this conception as suitable to the narrow purposes of logicians, he also insists that theoretical philosophy must have a richer conception of logical form: *it must be a formal specification of the cognitive faculty itself*. The forms of judgment, then, are ultimately forms of knowledge, to which certain contents are peculiarly suited.<sup>30</sup> (They are regarded, he says, “not merely according to inner validity, but also as knowledge in general” [KrV, A 71/B 96].<sup>31</sup>) It is apparently this conception that is the source of the Table's distinctive features.

Consider the example of the affirmative and infinite forms of judgment, which Kant distinguishes with respect to the “quality” (*Qualität*) of judgments. In traditional logic (he says) these would have been treated as equivalent, since they enter into inference forms in the same way. It makes no difference, with respect to inferences, whether the predicate concept is construed as positive or negative in itself. But it does make a difference with respect to the knowledge which judgments express. It is not just that someone will typically assign differential value to positive predicates (in the sense that it is more interesting to know what something is than what it is not); the knowledge itself will have a different formal character. While affirmative judgments express knowledge through a positive determination of the subject concept, infinite judgments are “limitative only” (*bloß beschränkend*, KrV, A 73/B 98) – they merely exclude a particular determination

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**29** This characterization of traditional logic is not completely accurate. Early modern logic textbooks were a hodgepodge of commonly recognized forms of judgment described against the background of a more systematic account of the roles of some of them in commonly recognized inference forms. Perhaps Kant's point is not that logicians were ignorant of any forms of judgment not connected with the syllogistic forms, but that they had no systematic justification for including them in logic. (This, at any rate, seems to be closer to the truth.)

**30** Here and everywhere I am translating Kant's word “*Erkenntniß*” as “knowledge”. This is imperfect for at least two reasons. First, the English word “knowledge” suggests a state of mind, or a disposition, while for Kant *Erkenntnis* is almost always active; an *Erkenntnis* is an act of knowledge (I myself will make much of this). Second, Kant indicates that *Erkenntnis* can be false (see KrV, A 58/B 83), while “knowledge” is usually understood as true. For these and other reasons recent translators have preferred to render “*Erkenntniß*” as “cognition”. That is understandable. (It is also supported by KrV, A 320/B 376.) Nonetheless, I prefer “knowledge” mainly because I do not find the word “cognition” a suitable alternative. To ordinary English speakers it has no meaning at all; but even to specialists it suggests something strange and technical.

**31** “[...] nicht bloß nach seiner innern Gültigkeit, sondern auch als Erkenntniß überhaupt [...]”

from the predicate through which the object is represented. (Consequently, Kant will associate the former with the category of “reality” [*Realität*], the latter with the category of “limitation” [*Limitation*].)

2. It can seem as though these considerations do not bear on the formal modalities, for judgments that differ in formal modality will also differ in their behavior in forms of inference. Kant says that “both judgments, the relation of which constitutes the hypothetical judgment [...], and likewise in whose reciprocity the disjunctive judgment consists [...], are in their entirety only problematic”;<sup>32</sup> also: “in a hypothetical syllogism, the antecedent occurs in the major premiss problematically, in the minor assertorically [...]” (KrV, A 75–76/B 100–101).<sup>33</sup> (He does not mention apodeictic judgments in this connection, but perhaps he takes them to correspond to conclusions of categorical syllogisms.<sup>34</sup>) For this reason, Longuenesse (among others) has concluded that “the modality of a judgment is determined by its relation to the forms of thought involved in deductive reasoning (judgments and syllogisms).”<sup>35</sup>

Clearly, there is a sense in which she is correct. The modality of a judgment can be taken to correspond to the manner in which it enters into forms of inference. It is important, however, not to overstate the significance of this. In particular, it is not an explanation of (M) – for it does not show why the “relation to the forms of thought involved in deductive reasoning” must be conceived *modally*, rather than in terms of giving and withholding assent. (Why, for example, could the relation of categorical predications to the form of hypothetical syllogisms not

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32 “[S]ind die beiden Urtheile, deren Verhältniß das hypothetische Urteil ausmacht [...], imgleichen in deren Wechselwirkung das disjunctive besteht [...], insgesamt nur problematisch.”

33 “[I]n einem hypothetischen Vernunftschluß das Antecedens im Obersatze problematisch, im Untersatze assertorisch vorkommt [...]”

34 I mean categorical syllogisms in the paradigmatic sense, the sense which issues in pure rational knowledge. (Kant does not deny inference a role in empirical knowledge – e.g. the inference that it is raining from the observation that everyone outside has an umbrella. But theoretical philosophy is not principally concerned with this kind of inference [see KrV, A 322/B 379].) In fact – because the apodeicticity of a conclusion can depend on the content of the inference, apodeictic judgments will present interpretive problems for those who take formal modality to have significance primarily in connection with forms of inference in general. (Thus A.O. Lovejoy says that “Kant’s conception of modality [...] does not properly permit of the classification of apodeictic judgments in the same scheme with the problematical and assertoric”. “Kant’s Classification of the Forms of Judgment”. In: *Philosophical Review* 16 (6), 1907, 601.)

35 *Op. cit.*, 159; cf. Longuenesse: *Kant on the Human Standpoint*. Cambridge 2005, 99, 190f.; Vuillemin, Jules: “Reflexionen über Kants Logik”. In: *Kant-Studien* 52, 1960–61, 322f.

be specified by observing that assent is withheld from the antecedent in the major premiss, and given in the minor<sup>36</sup> This is how it typically *was* specified in traditional logic.) In order to explain the relation of modalities to the forms of inference, there must first be an account of the modality of judgments in general.

### III

The main feature of this account, as I understand it, is described toward the end of section nine:

[E]verything [...] is incorporated into understanding stepwise, so that something is first judged problematically, then perhaps is also accepted assertorically as true, and finally is asserted as inseparably bound up with understanding, i.e., as necessary and apodeictic [...] (KrV, A 76/B 101).

– and, he says, it is this which explains why the formal modalities have to be included in the Table of Logical Forms. (His exact words are: “*Since* now everything here is incorporated into understanding stepwise [...] these three functions of modality can *therefore* be called so many moments of thinking in general” [emphasis added].<sup>37</sup>) It is the only explanation he gives for their inclusion.

The passage is compressed, but the point seems to be that the formal modalities, taken together, constitute a kind of progress in understanding – one typified by the order of modalities in a hypothetical syllogism, perhaps, but which also is a general feature of judging. The modality of a particular judgment would then

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**36** One answer would be: Kant already construes “the forms of thought involved in deductive reasoning” in modal terms. As a matter of fact I think does construe them this way (perhaps that is what Longuenesse has in mind; it is, at any rate, a point which has been suggested by some Kant interpreters – most notably Robert Brandom. (*Making It Explicit: Reasoning, Representing, and Discursive Commitment*. Cambridge, MA 1994, 7–13). But Kant cannot construe them this way *ab initio*: the construction is not self-evident; nor does it have a basis in any system of logic prior to the *Critique*.

**37** “Weil nun hier alles sich gradweise dem Verstande einverleibt, so daß man zuvor etwas problematisch urtheilt, darauf auch wohl es assertorisch als wahr annimmt, endlich als unzertrennlich mit dem Verstande verbunden, d.i. als nothwendig und apodiktisch, behauptet, so kann man diese drei Functionen der Modalität auch so viel Momente des Denkens überhaupt nennen.” Some commentators have thought that Kant means to say something special about the formal modalities in calling them “moments of thinking in general”. That is wrong. Many other passages (e.g. KrV, A 78/B 93) suggest that he regards all the forms of judgments as “moments of thinking in general”. Indeed, he thinks that a moment of thinking in general just is a form of judgment.

correspond to its place in this progress. Moreover, in view of the at least implicitly self-conscious character of judgment, as Kant conceives of it (KrV, B 131 et passim), the modalities at least implicitly represent this progress.

I want to draw two implications from this. First, *the modal values persist from one to the next*. The same possibility which is represented in a problematic judgment is also represented in an assertoric judgment; the same actuality which is represented in an assertoric judgment is also represented in an apodeictic judgment. Actuality includes possibility; necessity includes actuality.<sup>38</sup> The progress of modalities is not, therefore, a change of mind. In this respect, it is not like going from withholding assent to giving it.<sup>39</sup> Second, *the lower values anticipate the higher*. The possibility that is represented in a problematic judgment is not merely contrasted with actuality and necessity – as, for example, affirmation is contrasted with negation. The problematic judgment already includes the representation that it is the first stage in a progress culminating in apodeictic judgment. It does not just “fetch up” there, as if by accident. Apodeicticity is in view from the beginning, as the constitutive purpose of the judgment. In what follows, I will register this point by conceiving of the modalities in terms of the Aristotelian distinction between capacities and acts.<sup>40</sup> A problematic judgment I conceive as (and, as representing, at least implicitly) a capacity for the corresponding assertoric judgment; an assertoric judgment I conceive as (and, as representing, at least implicitly) a capacity for the corresponding apodeictic judgment.

Kant himself does not put things quite this way, and I could be accused of imposing a certain metaphysic on the text. As a general point, though, he conceives

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**38** Consequently, the disjunctions in (M) can also be taken inclusively. This is not an ambiguity in the account. It is a question of emphasis. If the formal modalities are construed as logical classifications of judgments, it will be natural to emphasize their distinctness. It would be confusing – not to mention tedious – if a single judgment had to be classified as problematic and assertoric and apodeictic. If, however, the formal modalities are taken as moments of the logical form of judgments, then it will be natural to emphasize their progressive unity – to speak of a “problematic moment” within assertoric judgment, and an “assertoric moment” within apodeictic judgment.

**39** It is natural to think that, when assent is given after being long withheld, this too is a kind of progress, for the subject must have learned something. This may be right. The point, however, is that assent does not represent progress. The fact that it can be taken as an indication of progress depends on the psychological history of the judgment in question, among other things.

**40** What is the alternative? Possibility and necessity, at least, might be conceived as representing mere conceptual properties, like non-contradictoriness, or having-a-contradiction-as-a-negation. One difficulty with such a conception is that it is unclear how progress could, in general, be effected from the former to the latter. Another is that, in Kant’s view, not all apodeictic judgments have contradictions as their negations. Only the analytic ones do.

of understanding as a “faculty” (*Vermögen*) whose act is judgment. What I am proposing is that these acts can themselves be divided into capacities and acts. (This will not lead to profligacy into the account so long as the further capacities are not construed as “distinct Agents”, in Locke’s word, each of which would have a distinct explanatory role in knowledge.<sup>41</sup>)

2. Before I say more, I want to head off a misunderstanding. “Progress in understanding” might suggest a process of learning. The problematic judgment would then be a kind of conjecture or hypothesis concerning a given object; the assertoric judgment would be a view of the hypothesis as true or false; the apodeictic judgment would be a sufficient explanation of this view.

This fits a common interpretation – for example, that of Jill Vance Brooker:

In problematic judgments one thinks or apprehends the judgment without making a commitment to a truth value [...]. Both assertoric and apodictic judgments involve assertions [...];<sup>42</sup>

or of Henry Allison:

[...] problematic [judgments] are deemed capable of a truth value, though, as problematic, this value is undetermined. By contrast, the truth value of [assertoric judgments] is determined [...].<sup>43</sup>

The difficulty, however, is that the tendency of human subjects to learn does not seem to explain why it is logically necessary to proceed stepwise through the formal modalities. It does not seem to be a logical fact that, before a predication can be represented assertorically, it must first have been represented problematically. Nor that before a predication can be represented apodeictically, it must first have been represented merely assertorically.

All of which has tended to make Kant’s theory of formal modality look like a kind of “psychologism”. Here is how Norman Kemp Smith puts it:

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<sup>41</sup> This is from the *Essay*: “For it being asked, what it was that digested the Meat in our Stomachs? It was a ready, and very satisfactory Answer, to say, That it was the digestive Faculty. What was it that made any thing come out of the Body? The expulsive Faculty. What moved? The Motive Faculty: And so in the Mind, the intellectual Faculty, or the Understanding, understood [...]” [Book II, Chap. XXI, § 20.]

<sup>42</sup> Brooker, Vance: *Kant’s “Critique of Pure Reason”: An Introduction*. Cambridge 2006, 91.

<sup>43</sup> Allison, Henry: *Kant’s Transcendental Idealism: An Interpretation and Defense* (second edition). New Haven 2004, 145f.

Kant's remark [concerning the progress of modalities] is irrelevant and misleading. The advance from consciousness of the problematic, through determination of it as actual to its explanation as necessary, represents only a psychological order in the mind of the individual.<sup>44</sup>

And this is from William and Martha Kneale's *Development of Logic*:

[Kant] says that the three 'functions of modality' which he distinguishes represent three 'moments of thought' which form a series. If this is indeed his doctrine, it is a bad instance of the corruption of logic by psychology and epistemology.<sup>45</sup>

It would indeed be "bad". It would be bad even as psychologism: for there does not seem to be anything in psychology to suggest that human beings typically proceed by judging, e.g. "[The predication is merely possible:] this S is P" ... "[The predication is actual:] this S is P" ... "[The predication is necessary:] Every S is P". (After all, some things are just obvious.) But I think another interpretation of KrV, A 76/B 101 is possible – one which takes account of the tendency of human subjects to learn, but which also construes the progress of modalities as an aspect of logical form rather than a psychological tendency.

## IV

1. In *Metaphysics* Θ, Aristotle distinguishes two senses in which a capacity is said to act.<sup>46</sup> In a first sense, it involves what he calls a "motion".<sup>47</sup> His example is the process of losing weight, i.e. an act of the body's capacity for alteration under certain conditions (e.g. diet and exercise) (Θ.6, 1048b19ff.). The word "alteration"

<sup>44</sup> Kemp Smith, Norman: *A Commentary to Kant's "Critique of Pure Reason"*. New York 1918, 194.

<sup>45</sup> Kneale, William and Martha: *Development of Logic*. New York 1985, 356.

<sup>46</sup> As Michael Frede has noted, the Greek terms "*dunamis*" and "*energeia*" are very difficult to translate into English. The former might, for example, be called a "potentiality", the latter an "actuality". I have preferred to speak of the "act of a capacity" in order to emphasize that *vépyeia* is something active. ("Actuality" can suggest "whatever is the case". Of course, for Aristotle *vépyeia* is also what is the case; the whole point is that this is, or at least can be, something active.) I have preferred to translate "*dunamis*" as "capacity" because the English word "potentiality" does not ordinarily convey something which can be active or inactive.

<sup>47</sup> "*Kinesis*". Not all *kineseis* are physical motions; he will use the term also for what might be called psychological motions. Cf. *Physics*, Book 3. For discussions, see Frede, Michael: "Aristotle's Notion of Potentiality in *Metaphysics* Θ". In: Scaltsas, Theodore, Charles, David and Gill, Mary Louise: *Unity, identity, and explanation in Aristotle's metaphysics*. New York 1994; Kosman, L. A.: "The Activity of Being in Aristotle's *Metaphysics*". *Loc. cit.*

itself suggests this: it is not a matter of a fatter thing's going out of existence and a thinner thing's taking its place; it is a matter of a thing which was fatter now becoming thinner.<sup>48</sup>

But – though Aristotle notes that “it is [widely] believed that an act is motion most of all” [Θ.3, 1047a32f.] – he does not think that motion is an act in the primary sense. In the primary sense, an act is not motion at all, for it involves no alteration. “For example, at the same time we are seeing and have seen [...] are happy and have been happy [...] are living and have lived” (Θ.6, 1048b22ff., emphasis added). These examples are not univocal, but the point seems to be that an act in the primary sense is distinguished by being its own end (so that, e.g. the act of seeing is at the same time for the sake of seeing). By contrast, motions always pertain to some further end, the achievement of which will bring the process to a natural close (so that the weight loss will end when the excess weight is lost). In ontological terms: if a motion is a process by which a thing is altered, an act in the primary sense consists in the capacity's simply being what it is. (In some cases it will be natural to say that it consists in the capacity's being more fully what it is.) It will be convenient to refer to acts in the primary sense as “pure acts”, and to use the term “act” (*simpliciter*) for the genus governing both motions and pure acts. (The capacity for a pure act I will call a “pure capacity”).

There are many questions about Θ.6 which I cannot address here. I only want to note, in connection with the interpretation of Kant, two uncontroversial facts. First, among the examples of motions which Aristotle gives – losing weight, walking, building, and so forth – he also includes learning (*manthanein*). For learning, as the acquisition of a new cognitive capacity, is an alteration in the soul. (This does not mean that learning is not in the soul's nature. Certain motions can, presumably, be natural for a thing.)

And second, Aristotle includes among his examples of pure acts knowing.<sup>49</sup> It seems to follow that to know – and, I would add, to judge – is not an alteration in understanding, but simply its being what it is.<sup>50</sup> (One indication of this is that – unlike learning – knowing does not presuppose an opposite state. Someone who knows may never have been ignorant. Someone who learns must have been.)

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<sup>48</sup> Kant makes a point of distinguishing “alteration [*Veränderung*]” from “change [*Wechsel*]” at KrV, A 187/B 230f. What is altered (paradigmatically, a substance) survives; what changes (paradigmatically, a state of a substance) does not.

<sup>49</sup> The terms he uses in Θ.6 (1048b24) are “*noein*” and “*phronein*”, which suggests that he means the point to apply to understanding in the broadest sense, both theoretical and practical.

<sup>50</sup> This does not mean that any particular knowledge could not be perfected in innumerable ways.

2. These considerations have been programmatic, but I think they suggest an alternative interpretation of the progress represented by the modalities: viz. as pure acts.<sup>51</sup>

On this interpretation, the progress from problematic to assertoric to apodeictic will not, in general, consist in alterations or, in particular, in learning. As I said, judging is closely connected with learning. It depends upon learning; it is, as might be said, the final cause of learning. But it itself is not learning. This has two important implications.

First, the progress represented by the modalities will not be intrinsically temporal. I do not mean that it will occur “outside of time” altogether. Clearly it does occur in time in the sense that it occurs under broadly temporal conditions: it comes to be in time, it is connected with the representation of temporal phenomena. But the progress itself does not take time. It is a progress in understanding alone.

This can be brought out, roughly, by considering why motions do take time. Motions consist in alterations. But alterations, as Kant understands them, are

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<sup>51</sup> Kant does not seem to have read much Aristotle, and it is likely that he did not know *Metaphysics* Θ. I am not claiming, however, that he consciously adopted Aristotle’s approach to knowledge. I am only claiming that the comparison with Aristotle’s approach is instructive. Still, it would be surprising if Kant had hit on Aristotelianism completely by accident – so it seems worth remarking on the ease with which German philosophy in the 18th century could draw on concepts from the Greek tradition.

Klaus Reich has put it very well: „[I]m 18. Jahrhundert stand es aus guten Gründen um die Philosophie in Deutschland besser. Was wir in Jahrtausende zurückliegender Vergangenheit und in den Ländern des Mittelmeeres suchen, wenn wir uns von der natürlichen Roheit und Blindheit befreien wollen, besaß man: wenn auch nicht in adliger Gestalt, so doch in ehrbarer und gesunder. Perücke und Zopf verhindern nicht, daß der Schulmeister den Lehrling zu einem tüchtigen Gesellen machen kann. Was Wolff und Baumgarten verbunden mit Lukrez und Seneca geben können an wirklichem Gehalt der Philosophie: wir sehen es am jungen Kant. [...] Ein Mann, der bei den Leibnizianern und den Römern in die Schule gegangen ist, kann die Griechen eher entbehren als wir, die wir beide nicht lesen. Er kann es. Aber andererseits glauben wir, daß unter solchen Voraussetzungen eine auch nur flüchtige und vermittelte Begegnung mit den Urhebern der Philosophie andere Wirkungen zeitigen muß, als es ohne sie der Fall ist. Begegnen Platon oder Panaitios einem philosophisch geübten, routinierten und kritisch gewordenen Geist auch nur einen Augenblick, so können sie anders begeistern, als uns, die wir sie zunächst literarisch und historisch beurteilen. Wir stehen ihnen als Laien gegenüber, Kant als Fachmann. Wir stehen zu ihnen als Liebhaber der Griechen, Kant als Kenner und Meister der Philosophie. Das bedingt eine andere Fähigkeit des Urteils, bedingt einen anderen Bedarf an Daten für das Urteil. Der Bankrotteur erlangt unter anderen Bedingungen Kapital als der Inhaber eines soliden Geschäftes. Auch hier garantiert nichts mehr die Erzielung von Gewinn als erzielter Gewinn. (Reich, Klaus: *Kant und die Ethik der Griechen*. Tübingen 1935, 3f.) – I am grateful to Michael Thompson for bringing this passage to my attention.



transitions between opposing states of a thing.<sup>52</sup> One state begins to be; the opposing state ceases to be.<sup>53</sup> Because they are opposed to one another, they cannot coexist. Assuming, then, that alterations take place under broadly temporal conditions – that “coexistence” means “temporal coexistence” – they must take time. Kant puts it like this:

[E]very alteration has a cause which shows its causality in the whole time in which it proceeds. Therefore this cause does not bring forth its alteration abruptly (at once or in one moment) but in time [...]<sup>54</sup> (KrV, A 208/B 253–254, cf. A 37/B 53–54)

With pure acts it is different. Since here the act just is the capacity which acts, it does not have to take time – even under broadly temporal conditions. (Imagine someone asking “How long does it take to know that?” It would be assumed he was asking how long it takes to learn it.)

There can of course be a considerable lag between the acquisition of a pure capacity (e.g. a problematic judgment) and the pure act (the assertoric or apodeictic judgment). But they can occur simultaneously. Evidently that is the case with all analytic judgments, which are immediately grasped as apodeictic, and with immediate inferences, or “inferences of understanding”, e.g. “Some bodies are heavy” as inferred from “Every body is heavy” (cf. Log §§ 44–55, AA 09: 115–119). And it seems to be the case with certain perceptual judgments, e.g. “This cinnabar is red”.<sup>55</sup>

Second, progress in understanding will not consist in the “determination” of a judgment.<sup>56</sup> The word “determination” is sometimes used loosely to mean any representation of a representation, but Kant means something more specific by it.

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**52** Properly speaking, the faculty of knowledge is not a “thing” for Kant in the sense that empirical objects are (cf. KrV, A 348ff.). But it is like a thing in a number of respects, as Matthew Boyle has pointed out (manuscript), and for these purposes can be treated as such without paralogism.

**53** This does not rule out that the opposing state should begin to exist in a weak sense as the initial state comes closer to completion. For example, Aristotle says, “Those who learn [something] at first do not yet know [it] [...] for it must become natural for them, and this requires time” (*Nicomachean Ethics*, 1147a22f.). But alteration can be understood as continuous so long as the extent to which a thing is in one state is inversely proportional to the extent to which it is in the opposing state. (In fact that is how Kant thinks of it [see KrV, A 208/B 254].)

**54** “[...] jede Veränderung [hat] eine Ursache, welche in der ganzen Zeit, in welcher jene vorgeht, ihre Causalität beweiset. Also bringt diese Ursache ihre Veränderung nicht plötzlich (auf einmal oder in einmal oder in einem Augenblicke) hervor, sondern in einer Zeit [...].”

**55** Cf. GMS II, AA 04: 420: “[I]f I think of a categorical imperative, I know immediately what it contains” (“Denke ich mir [...] einen kategorischen Imperativ, so weiß ich sofort, was er enthalte”). He refers to this imperative as “apodeictic” (*op. cit.*, 415).

**56** Irad Kimhi helped me to see this, and many other things.

“A determination,” he says, “is a predicate which is added onto the concept of the subject and enlarges it” (KrV, A 598/B 626). For example, in the judgment “Every body is heavy” the concept of a body is determined by the predicate heavy. In this sense, all – and only – synthetic judgments are determinations of concepts. But the pure act of these judgments – their assertoric modality – is not a determination of the problematic modality. There is indeed a temptation to speak as though it were: for it seems that something, some further predicate or predication, must have been added in order to explain the “actualization”. But nothing is – that is precisely the point. (“The actual contains no more than the merely possible”, is how Kant puts it [KrV, A 599/B 627].<sup>57</sup>) The only difference between a problematic judgment and an assertoric judgment is that, while the problematic judgment is a merely possible assertion, the assertoric judgment actually is one.

3. It is in the nature of a pure capacity to be purely active; for in being purely active, it is “more fully itself”, and it is in its nature to be itself. (“There is,” as Leibniz puts it, “a certain urge for existence or [so to speak] a straining toward existence in possible things or in possibility or essence itself; in a word, essence in and of itself strives for existence”.<sup>58</sup>) This does not mean that every pure capacity is always active. Obviously, not everyone who can see is always seeing; not all problematic judgments are assertoric judgments. But – here is the point – if a capacity is not active this is never because of something internal to it. Capacities, as such, act. It is never necessary to provide a special explanation for a particular capacity’s act. It is enough to say that it is a capacity.

Conversely, a special explanation will be necessary for a capacity’s inactivity. With problematic judgments, this explanation will typically involve a limitation within the subject himself. For example, if he is (as Kant puts it) “stupid” or “dull-witted” – or more generally, if his powers of judgment are limited in relation to his purposes in judging (KrV, A 133 n./B 172 n.)<sup>59</sup> – he may acquire the capacity to judge that, e.g. every metal is a body, without sufficiently distinct awareness of having done so. Or – having acquired the capacity – he may simply err in applying it (see Log § VII, AA 09: 53). Perhaps other explanations are possible. Perhaps a different kind of limitation explains why assertoric judgments are not apodeictic.

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<sup>57</sup> The remark comes in the specific context of a discussion of rational theology rather than logical modality, but its significance seems to be completely general.

<sup>58</sup> “De Rerum Originatione”. In: *Die philosophischen Schriften von Gottfried Wilhelm Leibniz*, Vol. VII. Ed. by C. I. Gerhardt. Berlin 1875–90, 303.

<sup>59</sup> The word “stupidity” – which is Kant’s (*Dummheit*) is misleading. Even if some subjects are genuinely stupid, any finite power of judgment will eventually run up against its limits; everyone is “stupid” to some extent.

(One kind of explanation which is not indicated, however, is that the empirical data might be insufficient to warrant judgment. I take it that the acquisition of the capacity presupposes all the extrinsic conditions sufficient for its act. Otherwise, it would not be a capacity for that act, but only a necessary condition of it.)

This points to a way of understanding the progress Kant describes at KrV, A 76/B 101: viz. as the gradual removal of limitations on the act of judging. If these limitations are expressed by the word “merely”, the progress could be expressed by leaving it off: as a progress from merely possible judgment to possible judgment without qualification, and from merely actual judgment to actual judgment without qualification. That is how an apodeictic judgment can be knowledge in the strictest sense and yet contain nothing more than the problematic judgment from which it emerged.

## Conclusion

1. When I began, I suggested that the theory of modality Kant introduces in the Critique of Pure Reason might be understood in connection with the “critical” conception of knowledge more generally. Since I have hardly said anything about the more general conception, I cannot say much about the connection. But – in closing – I would like to say a little.

2. The theory of formal modality is a theory of the formal act of judging. This does not just mean that the modalities which the theory is about are modes of that act. It also means that the theory is a theory of judging itself, as a formal act. Since judgments are formally distinguished from predications by their inclusion of modalities, it is part of the concept of judging in general that it includes the representation of itself as at least a capacity for a certain act. (It can even be said, I think, that this representation constitutes the form of judgments as such.) The progress in understanding which Kant describes is not, therefore, just one kind of progress which understanding makes among others; insofar as understanding is the faculty of judgment, it is the progress of understanding itself.

This suggests two points, which I take to be connected with the “critical” conception of knowledge more broadly. First, it is in the nature of understanding that it comes to know – and to know apodeictically. It is, in other words, fundamentally a faculty for knowledge a priori. This does not mean it always achieves apodeictic knowledge. For the most part it does not. But it always has apodeictic knowledge in view.

Second – it is in the nature of human understanding that its knowledge is limited in characteristic ways. (These limitations are what Kant has in mind when

he describes this understanding as “discursive” rather than “intuitive” [KrV, A 68/B 93].) Problems have to be thought through, insights gleaned, connections discovered – syntheses enacted (see KrV, B 139). It is the possibility of this kind of limitation, at two stages in the progress, which yields the three moments of formal modality in their special order: the problematic, the assertoric and the apodeictic. And it is this possibility which explains why they all have to be included in the Table of Logical Forms.

The two points – that understanding is a faculty of knowledge a priori, and that human understanding is limited in characteristic ways – will, I think, be recognized as distinctive themes of the Critique. In a way, they even constitute its fundamental insight: that human understanding is a faculty of synthetic knowledge a priori.<sup>60</sup>

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