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The Taylor instability in a rotating fluid confined between two cylinders has been investigated by light scattering. For rotation rates f asymptotically close to the critical rotation rate f_c, we find that the amplitude of the ordered flow varies as $(f-f_c)^{0.50\pm0.03}$. The axial structure of the ordered flow is sinusoidal near f_c, but harmonics become substantial for $f-f_c$ large. The main features of the Landau approach to hydrodynamic instabilities are thus confirmed.

A number of recent papers[1-6] have discussed the behavior of fluids near hydrodynamic instabilities. One motivation for this work is the suggestive analogy between fluid instabilities and second-order phase transitions, both of which exhibit an order parameter that grows from zero in the neighborhood of a critical point. Another is the hope of understanding the transition to turbulence, which occurs by means of a succession of progressively more complicated instabilities. Most of the previous work has been concerned with the Rayleigh-Bénard or convective instability. We have performed an experimental study of the Taylor instability[7] in a rotating fluid, and find excellent agreement with the Landau picture[8,9] of hydrodynamic instabilities.

The Taylor instability occurs when a fluid is confined between an outer stationary cylinder and an inner rotating one. If the rotation rate f exceeds a critical value f_c, the radial pressure gradient and the viscous forces are not sufficient to provide the required centripetal acceleration of the fluid, and a new flow pattern perturbs the z-independent Couette flow. Superimposed on the original azimuthal flow $V_\theta(r)$, there is now (Fig. 1) a toroidal roll pattern much like that of the Rayleigh-Bénard instability. Near f_c, the velocity components V_r and V_z are of course quite

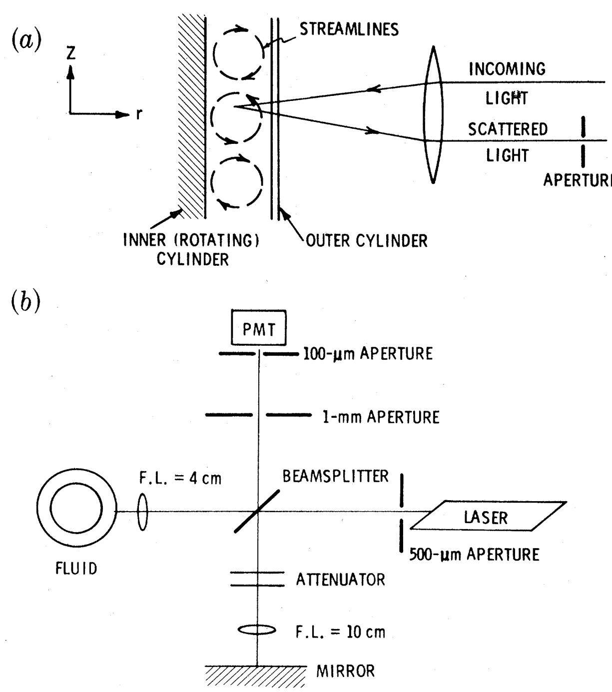

FIG. 1. (a) Side view of the cylindrical apparatus and light paths. The flow pattern shown schematically is actually superimposed on a much faster azimuthal flow perpendicular to the page. The axis of the rotating cylinder is along the z direction. (b) Top view showing the method used to mix the scattered and reference beams. Each arm is 30 cm long.

small. If f is increased considerably beyond f_c, there are further instabilities in which the vortices acquire circumferential waves, and eventually a transition to a turbulent state (nonperiodic in time) occurs. The first instability is the subject of this paper.

According to the Landau theory[8] and subsequent work[9] one expects that just above f_c, the radial velocity should be of the form $V_r = A_1(r, \epsilon)\cos(k_1 z)$, where $\epsilon = (f - f_c)/f_c$ and the amplitude A_1 varies as $\epsilon^{1/2}$. Fluctuations may influence this dependence[1,2] for $\epsilon < 10^{-5}$, but this region is probably not experimentally accessible. As ϵ is increased higher harmonics should appear as a result of the the nonlinear terms in the Navier-Stokes equations, so that

$$V_r = \sum_p A_p(r, \epsilon)\cos(pk_1 z). \quad (1)$$

The discussion above refers to the steady-state situation. The response time of the system when perturbed from equilibrium is expected to diverge as $\epsilon \to 0+$. The major new result of the present work is the measurement of $A_p(\epsilon)$ for $p = 1, 2$.

In order to study $V_r(r, z, \epsilon)$ we have utilized laser light (5 mW at 5145 Å) scattered at an angle of 171° from the forward direction by a dilute suspension of 2-μm polystyrene latex spheres in water, as shown in Fig. 1. The scattered light was mixed with an unscattered but attenuated beam by use of a Michelson-like interferometer, and the power spectrum[10] of the photocurrent was obtained from a real-time spectrum analyzer. The power spectrum exhibits a peak approximately 200 Hz wide at a frequency in the range 0 to 10 000 Hz. For our geometry the ratio of the local radial velocity to the mean frequency of the peak is 1.94×10^{-5} cm sec^{-1} Hz^{-1}. The scattering volume was experimentally determined to be about 1 mm long in the radial direction and 0.2 mm in the orthogonal directions.

The fluid was contained between an inner black aluminum cylinder of radius 1.555 cm, and a precision-bore Pyrex tube of inner radius 2.540 cm. The cylindrical region was 30 cm long, and was temperature controlled to within 0.03°C. Temperature control was necessary because f_c changes by 2% per degree because of the temperature dependence of the kinematic viscosity of the fluid. At 27.0°C, we found f_c to be 0.0651 ± 0.0002 Hz, which is consistent with the predictions of Chandrasekhar.[7] (A quantitative comparison is not possible because the linear stability theory has only been evaluated for special geometries in which the ratio R_1/R_2 of radii either is $\frac{1}{2}$ or approaches unity. However, previous work[11] has established the correctness of theoretical predictions for f_c and k_1.) The rotation rate was measured electronically to an accuracy of 0.02% and was constant to within 0.04%.

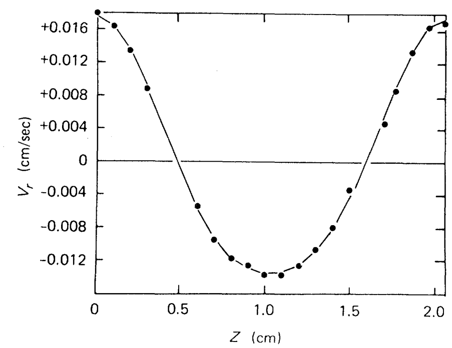

FIG. 2. Axial variation of the radial velocity for $(f-f_c)/f_c = 0.014$. The solid curve includes the fundamental term and a small second-harmonic term.

The z dependence of V_r was studied halfway between the inner and outer cylinders by translating the apparatus with the optics unchanged. In Fig. 2 we present the results for $\epsilon = 0.014$. The maximum value of V_r is only $0.06V_\theta$, so that the periodic structure is a fairly small perturbation on the azimuthal flow. As predicted for small ϵ, the flow is nearly sinusoidal. However, there is a small second-harmonic term, which can be detected by the difference in magnitude of the positive and negative peaks. The amplitudes of the fundamental and second-harmonic terms (obtained by computer Fourier analysis) are $A_1 = 0.0155$ cm/sec and $A_2 = 0.0019$ cm/sec, and the solid curve shows that these two terms alone produce an excellent fit, with a fundamental wave number $k_1 = 3.05$ cm^{-1}. The product $k_1 R_2$ is 7.75, which can be roughly compared to Chandrasekhar's prediction[7] $k_1 R_2 = 6.4$ for the case $R_1/R_2 = \frac{1}{2}$.

As ϵ becomes larger, the second and higher harmonics increase in importance relative to the fundamental term, because of their dependence on higher powers of ϵ. In Fig. 3, we present $V_r(z)$ for $\epsilon = 0.465$. Here the magnitude of V_r has a higher and sharper peak in the outward flowing regions than in the inward flowing ones. We find that three terms are necessary to produce a fit to within the accuracy of the experiment, and the amplitudes are $A_1 = 0.1030$ cm/sec, $A_2 = 0.0356$ cm/sec, and $A_3 = 0.0072$ cm/sec. The wave number k_1 is 3.20 cm^{-1}, and is independent of ϵ over

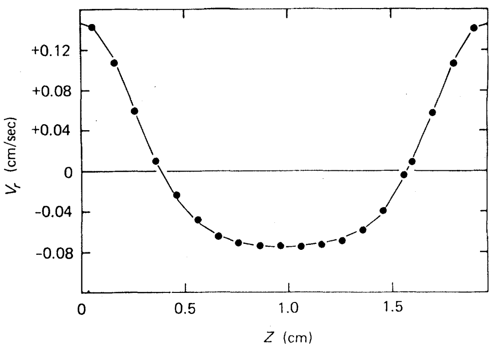

FIG. 3. Axial variation of the radial velocity for $(f-f_c)/f_c=0.465$. The solid line includes the fundamental and the second- and third-harmonic terms.

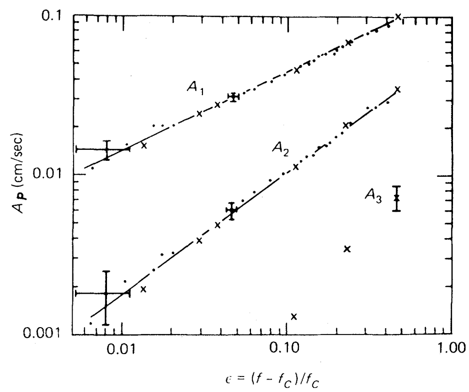

FIG. 4. Dependence of the Fourier coefficients A_p on the rotation rate. The solid lines represent the results of least-squares analyses, with $f_c = 0.0651$ Hz.

the range $0<\epsilon<0.5$ to within the precision of the measurements (5%). The precision is limited by small thermally induced drifts in the flow pattern near $\epsilon=0$.

The dependence of the Fourier coefficients A_p on ϵ is shown in Fig. 4. Points marked by a cross were obtained by fixing the rotation rate, allowing the system to equilibrate (which required at least 15 min when $\epsilon<0.01$), measuring $V_r(z)$, and then performing a complete Fourier analysis. The first two amplitudes A_1 and A_2 can also be obtained from a simpler procedure, in which only the magnitudes P_+ and P_- of the positive and negative peaks in $V_r(z)$ at each value of ϵ need be measured. We utilize the combinations $\frac{1}{2}(P_+ + P_-) = A_1 + A_3 + \ldots$ and $\frac{1}{2}(P_+ - P_-) = A_2 + A_4 + \ldots$ to accomplish this. The amplitude A_3 is at most 7% of A_1 in the range of interest, and can be removed as a small correction by using $A_3(\epsilon)$ as obtained from Fig. 4 and other data. Removal of A_4 is unnecessary, as it was found to be negligible over the range of ϵ studied. Points for which A_1 and A_2 were obtained in this manner are indicated by dots in Fig. 4. These points are consistent with those obtained by complete Fourier analysis of $V_r(z)$.

To test the prediction that A_1 varies as $\epsilon^{1/2}$, a least-squares analysis was performed, in which the weighted sum of the squares of the deviations from $D(f-f_c)^\beta$ was minimized with D, β, and (if desired) f_c as free parameters. The weighting was determined by assuming the measurement error ΔA_1 to be independent of ϵ. We determined experimentally that $0.0649<f_c<0.0653$ Hz by noting the rotation rate at which the photocurrent power spectrum no longer has a peak at $\nu>0$. The uncertainty arises partly from the fact that frequency shifts of less than 200 Hz are not detectable, and partly from what appear to be slight variations in f_c from run to run. We find that $A_1 = (0.145 \pm 0.013)\epsilon^{0.50\pm0.03}$ cm/sec, where the uncertainties are due mainly to the uncertainty in f_c. If we treat f_c as a free parameter, the best fit occurs when $f_c = 0.0649$ Hz and $\beta = 0.52$. We conclude that the measurements are consistent with the prediction $A_1 \sim \epsilon^{1/2}$. The data of Berge and Dubois,[4] who obtained $\beta > \frac{1}{2}$ for the convective instability, may have been influenced by the second and higher harmonics, which were not measured in their experiments.

Applying the same approach to $A_2(\epsilon)$, we found that $A_2 = (0.063 \pm 0.005)\epsilon^{0.77\pm0.03}$ cm/sec represents the data quite well. In order to make sure that we were actually observing asymptotic behavior, we repeated the analysis after eliminating points for $\epsilon>0.1$, and found the exponent to be 0.76 ± 0.06, unchanged except for a larger error. We have not considered A_3 in detail because the data are at present insufficient for an accurate determination of the ϵ dependence of A_3.

Our results confirm the Landau approach to fluid behavior asymptotically near an instability leading to a spatially periodic but time-independent flow. In particular, a single spatial Fourier component dominates near the critical point, and its amplitude grows as $\epsilon^{1/2}$. However, the exponent 0.77 ± 0.03 of the second-harmonic term, which becomes important farther from the critical point, is not understood at the present time. Hopefully this and other experiments will stimulate theoretical work aimed at understanding the nonasymptotic region.

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[1]R. Graham, Phys. Rev. Lett. 31, 1479 (1973).
[2]W. A. Smith, Phys. Rev. Lett. 32, 1164 (1974).
[3]H. Haken, Phys. Lett. 46A, 193 (1973).
[4]P. Berge and M. Dubois, Phys. Rev. Lett. 32, 1041 (1974).
[5]G. Ahlers, Phys. Rev. Lett. 33, 1185 (1974).
[6]J. B. McLaughlin and P. C. Martin, Phys. Rev. Lett. 33, 1189 (1974).
[7]See, for example, S. Chandrasekhar, *Hydrodynamic and Hydromagnetic Stability* (Clarendon Press, Oxford, England, 1961).
[8]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, London, 1959).
[9]For a review, see R. C. DiPrima and E. H. Rogers, Phys. Fluids, Suppl. II, 155 (1969).
[10]For a review of light-beating spectroscopy, see H. Z. Cummins and H. L. Swinney, in *Progress in Optics*, edited by E. Wolf (American Elsevier, New York, 1970), Vol. 8.
[11]R. J. Donnelly and D. Fultz, Proc. Roy. Soc., Ser. A 258, 101 (1960).

Breaking the Roton Barrier: An Experimental Study of Motion Faster than the Landau Critical Velocity for Roton Creation in He II†

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We report the first observation of objects moving through He II with equilibrium drift velocities $\bar{v}$ beyond the Landau critical velocity v_c for roton creation. With $T \sim 0.4$ K, $P = 25$ bar, $F = 2$ kV cm^{-1}, $\bar{v} - v_c$ for negative ions is larger than a theoretical prediction by a factor of $\sim 10^5$. The vortex-ring nucleation rate is found to decrease with F above 300 V cm^{-1}, thus resolving apparent inconsistencies between earlier experiments.

In his celebrated explanation of superfluidity, Landau[1] showed that the kinetic energy of a liquid flowing at velocity v through a tube (or that of a heavy object moving through the liquid) cannot be dissipated through the creation of an excitation of energy ϵ and momentum p in the liquid unless $v \geq \epsilon/p$. For He II the minimum value of ϵ/p is nonzero, occurring close to the roton minimum in the elementary excitation spectrum, so that a critical velocity $v_c = (\epsilon/p)_{\min} \approx 50$ m sec^{-1} exists, below which dissipation ought not to occur in the superfluid. Measured critical velocities are usually orders of magnitude smaller, because of the onset of vortex formation at lower velocities, but Rayfield[2] reported that the drift velocity $\bar{v}$ of negative ions in He II under pressure $P > 12$ bar below 0.6 K appeared to reach and to be limited by v_c when the applied field was raised to about 70 V cm^{-1}. This has remained the only known situation to which Landau's original criterion for the breakdown of superfluidity appears to be relevant.

Takken[3] has considered roton creation by negative ions moving at velocities slightly greater than v_c on the basis of a wave radiation model in which each ion is assumed to generate a conical wave of coherent roton radiation, much like the disturbance produced by an airplane breaking the sound barrier. By analogy with the aerodynamic case, a rapid increase in drag is expected as the velocity increases past v_c: Takken concluded that an upper bound on v is given by $v_{ub} = v_c(1 + 10^{-12}F^2)$, where the electric field F is in V cm^{-1}. A 1% increase of v above v_c would therefore require $F \geq 10^5$ V cm^{-1}, implying that any increase of v beyond v_c ought to be almost impossible to observe experimentally.

An attempt by Neeper and Meyer[4] to test this remarkable assertion was thwarted by an unexpected increase in the vortex nucleation rate ν with falling temperature, such that at 0.3 K only vortex rings, and no bare ions, arrived at their collector. Our recent observation[5] that the field emission current at 0.3 K in He II increases dramatically with P above 12 bar, and is temperature independent below 0.4 K, seemed to be inconsistent with Neeper and Meyer's result. Approximate values of ν deduced[6] from the field-emission measurements apparently indicated the feasibility of our present experiment to test Tak-