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External Noise and the Onset of Turbulent Convection

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The effect of fluctuations on the transition to turbulence is examined by introducing large-amplitude noise into the boundary temperature of a Rayleigh-Bénard system. The response can be largely described in terms of variables whose induced fluctuations are much slower than either the natural oscillations or the external noise. However, the sequence of transitional regimes and the statistical properties of the turbulent state are not dramatically affected.

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Nonhydrodynamic fluctuations sometimes influence the statistical behavior of a fluid near its transition to turbulence. For example, the level of random external disturbances or thermal noise determines the time scale of unstable oscillations in a recent model of convection in a rotating layer proposed by Busse and Heikes.[1] External disturbances dramatically influence the spectrum of velocity fluctuations in a turbulent free-shear layer.[2] Recent experimental studies of the onset of turbulent Rayleigh-Bénard convection by Ahlers and Walden[3] have been interpreted[3,4] as indicating that random forces may cause the time dependence which is observed when the aspect ratio (layer radius to depth) is sufficiently large.

In contrast to these examples, studies of the transition to turbulent convection at small aspect ratio[5,6] have not shown evidence that external or thermal noise observably influences the actual time-dependent motion. In the present paper, we report the results of experiments in which a small-aspect-ratio fluid layer is subjected to spatially uniform but broadband temperature fluctuations at the lower boundary. We find that the resulting modifications of the local velocity can be represented accurately by a finite Fourier series with time-dependent coefficients and phases. The induced fluctuations contain a linear part with a spectrum similar to the external noise, and a nonlinear part that is greatly enhanced at low frequencies. However, the sequence of regimes leading to turbulence and the statistical properties of the turbulent state are not substantially changed, indicating that the system is governed by intrinsic hydrodynamics when the boundary conditions are time independent.

The apparatus and laser Doppler methods used to determine the fluctuations in local fluid velocity have been described elsewhere.[5] The experiments utilize a cell with an aspect ratio (largest horizontal dimension to fluid depth) of 3.5 and are conducted at a Prandtl number of 2.5. The mean flow consists of two Bénard rolls oriented parallel to the short cell edge, throughout the duration of these experiments. (Other stable flows can be obtained by varying the initial conditions, as reported previously.[5])

The lower boundary temperature is modulated with noise by introducing computer-generated random voltages into the electronic feedback loop controlling the temperature. The equivalent rms fluctuation in R/R_c is typically 0.3, where R_c is the critical Rayleigh number of an infinite fluid layer. The temperature spectrum is not white because of the response time of the copper plate, but contains significant power over the entire bandwidth (0.5 Hz) of the velocity fluctuations which are normally observed near the onset of turbulence. The temperature fluctuations have a distribution function that is roughly Gaussian, and their total spectral power is less by a factor of at least 10^3 when the noise modulation is absent.

For time-independent boundary conditions, time-dependent (periodic) convection begins at $R/R_c = 20$. The basic frequency is initially about 0.07 Hz, but a subharmonic is present for $R/R_c > 21$. The spectrum of the velocity variations $v(t)$ then consists of sharp peaks at multiples of $f_0 = 0.034\,80$ Hz superimposed on an instrumental white noise background, as shown in Fig. 1(a). When the external noise is introduced, the time record becomes noticeably erratic [Fig. 1(b)]. The corresponding spectrum contains a low-frequency background, and the peaks are slightly broadened. These changes in the velocity spectrum, while measurable, are small in comparison with those that occur spontaneously as R is changed.

In order to study these induced fluctuations, we represent the data by a finite Fourier series with slowly varying amplitudes and phases computed

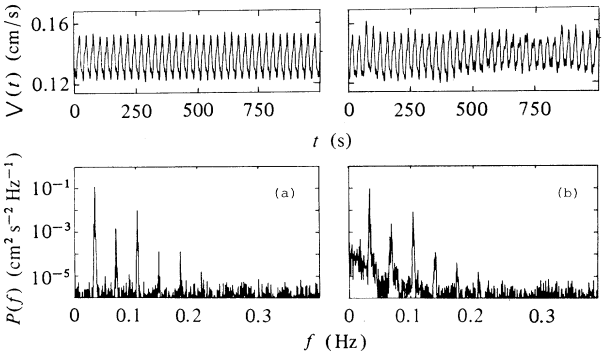

FIG. 1. The velocity $v(t)$ and corresponding power spectrum at $R/R_c = 27.5$ for (a) stationary boundary conditions and (b) noisy boundary conditions.

over successive intervals of length $P = f_0^{-1}$:

$$v(t) = C_0(t) + \sum_{n=1}^{N} C_n(t) \cos[2\pi n f_0 t + \varphi_n(t)].$$

In this approach the broadening of spectra arises from nonperiodic time dependence in the coefficients of a small number of basic oscillations. The introduction of slow variables is similar in spirit to the study of nearly periodic dynamical systems by examination of Poincaré sections.[7] The decomposition is meaningful because most of the power is concentrated at frequencies nf_0.

The time record and spectrum of Fig. 1(b) are almost perfectly duplicated (up to nearly $4f_0$) by a series containing 4 terms ($N = 3$). The induced fluctuations are shown in Fig. 2. Since the boundary temperature had been recorded simultaneously, we were able to compare the external noise with the induced mean fluctuations $C_0(t)$, and noted that the records were nearly the same (except for scale). Thus the fluctuations in $C_0(t)$ (averaged over the time interval f_0^{-1}) are largely linear in the perturbations. This is not surprising since, under static conditions, the mean velocity grows linearly for small increments of R/R_c. We determined the corresponding scale factor and subtracted these linear fluctuations from the velocity data in order to examine the time dependence of the higher-frequency terms more accurately. The amplitudes $C_1(t)$ and $C_2(t)$ are also shown in Fig. 2. Low frequency fluctuations are the dominant response to the external noise, and also dominate $C_0(t)$ after removal of the linear response.

To estimate the correlation time of the slow fluctuations in $C_n(t)$, we computed their spectra $P_n(f)$ and normalized them by the spectrum of

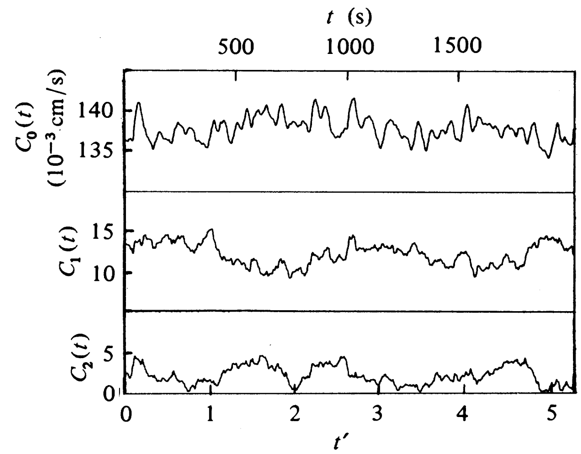

FIG. 2. Time dependence of the Fourier amplitudes $C_n(t)$ for the data of Fig. 1(b). The fluctuations in C_0 nearly follow the external noise, but the fluctuations in C_1 and C_2 last roughly one thermal diffusion time (one unit on the lower scale).

the external noise $P_{ext}(f)$ in order to compensate for its spectral shape. The integral

$$I(f) = \int_0^f [P_2(f)/P_{ext}(f)]df$$

is plotted in Fig. 3. The knee in the curve at about $f_c = 0.003$ Hz indicates that most of the spectral power in the induced fluctuations is at lower frequencies than f_c. It is interesting to note that f_c^{-1} is approximately equal to the thermal diffusion time $d^2/\kappa = 380$ s for this cell, where d is the depth and κ the thermal diffusivity. The cutoff frequency f_c is far lower than the characteristic oscillation frequency f_0. (Of course, our

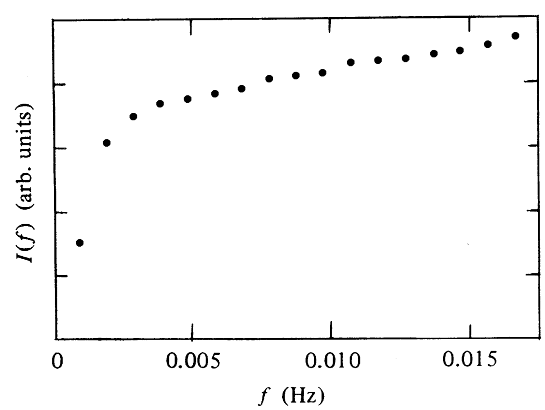

FIG. 3. The integral of the spectrum of $C_2(t)$, normalized by the spectrum of the external noise. The knee at 0.003 Hz indicates that the induced fluctuations are mostly at very low frequencies.

method of analysis would not be valid if f_c were comparable to f_0.)

Now we consider the effect of external noise on the transition to turbulence. We varied R in small steps over a period of several months, in each case measuring the time dependence of the velocity with and without external noise present. In order to summarize these results it is necessary to introduce some measure of the extent to which periodic spectra are made broadband by noise modulation. Following Crutchfield *et al.*,[8] we define

$$D=(\sum_{i=1}^{K} S_i)^2/\sum_{i=1}^{K} S_i^2,$$

where the S_i are the K (typically 1025) spectral estimates. If only one channel of the spectrum has significant power, $D \cong 1$, while a white-noise spectrum has $D \cong K$. This quantity adequately characterizes the relative amount of discrete and broad spectral structure unless the spectral resolution is not high enough. For example, $D = 2.15$ for Fig. 1(a) and $D = 2.90$ for Fig. 1(b).

The dependence of D on Rayleigh number for stationary and noisy boundary conditions is shown in Fig. 4. The spectra are nearly periodic when $D \leqslant 5$, but thoroughly broadband for $D \gtrsim 20$. In the periodic regime, the external noise slightly increases D by inducing fluctuations of the kind we have already described. (The rise in D near $R/R_c = 30$ is associated with a broadening of the spectral peaks that does not persist over a wide range in R.) Nonperiodicity occurs for $R > 41$ via

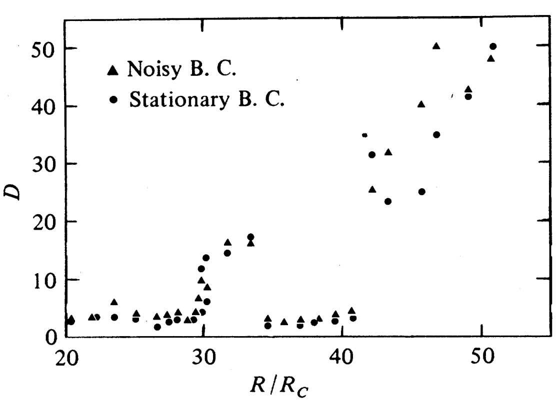

FIG. 4. The dependence of the parameter D, which measures the breadth of the velocity spectrum, as a function of R/R_c. The presence of external noise (triangles) slightly enhances D in the periodic regimes (small D), but produces no statistically significant change in the turbulent regime.

a quasiperiodic interval containing several apparently incommensurate frequencies. In the nonperiodic regime, the statistical fluctuations are large and uncorrelated with the presence of external noise.

Turbulent flows are believed to be unstable at each instant, in the sense that small (even thermal) fluctuations are amplified exponentially in time.[9] However, this sensitivity to initial conditions does not necessarily imply that the statistical properties of the spontaneous fluctuations are substantially influenced by noise. In this case, we find that even large-amplitude external noise produces relatively small effects on spectra of the velocity fluctuations, indicating that the turbulent dynamics and transitional phenomena are probably dominated by intrinsic effects under the conditions of our experiments.

The long-time properties of a randomly stirred fluid have received some theoretical attention.[10] However, our observations are perhaps more directly comparable to a recent investigation of a noise-forced Lorenz model by Zippelius and Lücke,[11] since models of this type are believed to have some relevance to small–aspect-ratio convection. The Lorenz model[12] does not show periodic oscillations, but the correlation time for noise-induced fluctuations is comparable to the thermal diffusion time in the conduction regime $R < R_c$. This behavior is qualitatively similar to our experimental observation of a time scale $f_c^{-1} \cong d^2/\kappa$ for the slow variables $C_n(t)$ in the oscillatory regime. Zippelius and Lücke also find rather small noise effects on time correlations in the turbulent regime, as we found experimentally. Other noise-forced dynamical systems[13] have been studied recently, but are not directly comparable to these experiments.

The fact that the induced fluctuations in the periodic regime are of very low frequency (comparable to the vertical thermal diffusion time, but slow compared to the periodic pretransitional oscillations) is interesting in view of the fact that the spontaneous onset of nonperiodicity proceeds by way of a quasiperiodic regime which introduces slow modulations of basic oscillations. Apparently, the low-frequency fluctuations characterize both the response to noise and the onset of turbulence.

In general, these experiments show that stochastic driving forces should not be invoked in models of the transition to turbulent convection at small aspect ratio. Rather, the weakly turbulent state can reasonably be regarded as a strange attractor in a low-dimensional dynamical system.[14] The situation at large aspect ratio is not yet clear.

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Holographic Imaging without the Wavelength Resolution Limit

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It is usually assumed in both optical and acoustical holography that the resolution of a reconstructed image is limited by the wavelength of the radiation. In this paper it is demonstrated that this is not necessarily true in acoustical holography. A technique which images the source vector intensity as well as the sound pressure amplitude with a resolution independent of the wavelength is presented. Application to electromagnetic radiation may be possible.

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It is usually assumed in holography that the spatial resolution in a reconstructed image is limited by the wavelength of the radiation. This is certainly true in optical holography and is usually assumed in acoustical holography, with the result that acoustical holography has not been used in otherwise natural applications such as low-frequency–noise abatement and musical-instrument research. However, the wavelength resolution limitation is not intrinsic to the fundamental theories of holography but rather is due to experimental limitations which are present in optical holography but are not necessarily present in acoustical holography. In this paper we demonstrate experimentally that it is possible to obtain high-resolution images of sound sources regardless of the wavelength, and that a single measurement is sufficient to reconstruct the source sound pressure, the particle velocity, the far-field radiation pattern, and most importantly the source vector intensity. Our results should form a basis for the development of powerful new tools for research in both acoustic and electromagnetic radiation, although the latter would require more consideration of the vector nature of the field.

The theory underlying non–wavelength-limited acoustic imaging is as follows: One assumes a monochromatic source region (radiating into a linear and homogeneous medium with a wavelength λ) located to one side of an infinite plane I defined by $z=z_I$. From Green's theorem, the sound pressure amplitude and phase in the plane I [represented by the complex quantity $P_I(x',y')$] can be used to calculate[1] exactly the sound field P at any field point (x,y,z) to the side of the plane away from the sources $(z>z_I)$:

$$P(x,y,z)=\iint P_I(x',y')G(x-x',y-y',z-z_I)dx'dy', \quad (1)$$

