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STIMULATED EMISSION OF THE 3.04-CM FINE-STRUCTURE LINE OF HYDROGEN IN DIFFUSE NEBULAE

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ABSTRACT

Intense Lyman-α radiation in diffuse nebulae can excite the 2 $^2P_{3/2}$ level of hydrogen to such an extent that the emission in the fine-structure line stimulated by the free-free continuum becomes observable For the Orion Nebula, the measured continuum flux, together with Pottasch's estimates of the 2 P population, implies an antenna temperature in the emission line of 0°02 K, if an 85-foot telescope of moderate efficiency is used. This value is at present difficult to detect, but equipment already being developed may ultimately achieve a signal-to-noise ratio of 100:1. Such observations will be significant for the information they give concerning the Lyman-α radiation field in nebulae.

In recent papers Pottasch (1960*a*, *b*) and Yada (1960) have emphasized the effects which trapped Lyman-α radiation can have on the observed spectrum of a diffuse nebula. Resonant excitation of the 2P levels can lead to a significant 2P population in the steady state. As a result, collisional 2P–2S transitions can augment the observed 2S–1S two-quantum emission, and the nebula may become opaque in the Balmer lines.

Yada considered the intensity of the $2^2P_{3/2}$–$2^2S_{1/2}$ fine-structure line at 9847 Mc/s (λ 3.04 cm) to be expected on the basis of his various models. However, he included only spontaneous transitions and therefore concluded that the line would be too weak for detection. We show below that stimulated emission is much more important, so that, at least on the basis of Pottasch's estimates of the 2P population (which are somewhat more definite than Yada's for the specific case of the Orion Nebula), there is a possibility of detecting the line in the Orion Nebula.

The idea of 3-cm line emission by diffuse nebulae originated with Wild (1952), who considered that the 2S would be more populated than the 2P, because of the metastability of the former level. Following him, Townes (1957) suggested that the line might be observable in absorption. His quantitative estimates were discouraging, however. Shklovsky (1960) pointed out that Lyman-α could in principle excite the $2P_{3/2}$ level so that the line would be seen in emission. He concluded (without detailed calculations) that it would be too weak to detect. The present position appears to be this: the 2S population, which can be estimated directly from the observed two-quantum emission intensity, is too low in most nebulae to produce an observable 3-cm absorption line. Therefore, the only hope of detecting the line arises from the possibility that the 2P is much more populated than the 2S, owing to the "pumping" effect of Lyman-α. As Pottasch states that $N(2P)/N(2S) \gg 1$, detailed estimates of the line intensity in the Orion Nebula (a favorable case) seem justified.

If the Lyman-α optical depth is large enough, Lyman-α photons will scatter many times (n) before escaping from the nebula. If n approaches $10^{12} N_e^{-1}$ ($\simeq 10^9$) in the Orion Nebula), there is a good chance that a Lyman-α absorption will one time be followed by a 2P–2S collisional transition rather than by the usual 2P–1S de-excitation. Since the 2S atom usually emits two quanta, dropping quickly to the ground level, this cycle destroys the Lyman-α photons. Under these conditions the equations of statistical equilibrium imply that

$$\frac{N_P}{N_S} = \frac{\varpi_P}{\varpi_S}\left[1 + \frac{A_{SG}}{C_{SP}}\left(1 + \frac{A_{CS}}{A_{CP}}\right)^{-1}\right], \tag{1}$$

where the A's are radiative and the C's collisional transition rates. Equation (1) is equation (13) of Pottasch (1960*b*) and is based on a four-level atom consisting of ground (G), 2S, 2P, and continuum (C) levels. As $A_{\mathrm{CS}}/A_{\mathrm{CP}}$ is of order unity and $A_{\mathrm{SG}}/C_{\mathrm{SP}}$ is of order $10^4\,N_e^{-1}$, one finds that, for small N_e, $N_{\mathrm{P}}/N_{\mathrm{S}}$ can exceed unity by a large factor. It should be emphasized that this result depends strongly on a sufficiently large value of n, which in turn depends on a solution to a rather difficult non-coherent transfer problem. Although many authors have presented solutions to this problem (Sobolev 1944; Zanstra 1949; Unno 1951; Field 1959; Yada 1960; Pottasch 1960), the great variety of approximations made indicates that the problem has not yet been solved definitively. According to Pottasch, an optical depth in Lyman-α equal to 10^6 should insure that the number of scatterings is sufficient that equation (1) may be applied to the Orion Nebula; on the other hand, Field estimated that 10^9 would be required. As the actual value of the Lyman-α optical depth is uncertain in any case, our procedure will be to assume that equation (1) is correct and to find the resulting 3-cm line intensity. Observations of the line can then be used to discriminate among the rather uncertain theoretical treatments of the Lyman-α transfer problem.

Starting from equation (1) and Pottasch's estimates of the electron density, we find $N_{\mathrm{P}}/N_{\mathrm{S}} = 26$ in the inner 2.4×10^{18} cm of the Orion Nebula, and 80 in the outer 7.1×10^{18} cm. The 2S populations are estimated by Pottasch to be 5.7×10^{-8} cm^{-3} (inner) and 4.4×10^{-9} cm^{-3} (outer) from the observed two-quantum intensity. Hence the 2P populations are 1.5×10^{-6} cm^{-3} and 3.5×10^{-7} cm^{-3}, respectively. Under excitation by Lyman-α, two-thirds of the 2P atoms are in the $2^2\mathrm{P}_{3/2}$ level.

As the 3-cm line is broadened primarily by radiation damping in the Lyman-α line (Wild 1952), the absorption coefficient at the line center (9847 Mc/s) is given by

$$\kappa = \frac{32\pi^3 S\nu}{3hc\Gamma}\left[\frac{N(\mathrm{S}_{1/2})}{\varpi(\mathrm{S}_{1/2})} - \frac{N(\mathrm{P}_{3/2})}{\varpi(\mathrm{P}_{3/2})}\right]\mathrm{cm}^{-1}, \tag{2}$$

where the line strength, S, is $36\,a_0^2 e^2$ and the effective damping constant, Γ, is 8.4×10^8 sec^{-1} rather than $A(\text{2P–1S}) = 6.2 \times 10^8$ sec^{-1} because of a slight hyperfine broadening (Wild 1952). As the second term exceeds the first in equation (2), stimulated emission dominates true absorption, and the resulting opacity is negative. Putting in the densities and distances stated above, we find that the opacity from the center of the Orion Nebula to its exterior is

$$\tau = -4.2 \times 10^{-3}\,. \tag{3}$$

The same 2P atoms, of course, will absorb the Balmer lines. One can show easily that

$$\tau(\mathrm{H}\alpha) = -713\,\tau(\lambda\,3\ \mathrm{cm}), \qquad \text{or} \qquad 3.1 \text{ for the Orion Nebula.} \tag{4}$$

We assumed that Hα is Doppler-broadened with an r.m.s. velocity of 9.1 km/sec, appropriate to 10000° K. As we are basing our discussion on Pottasch's estimates for the 2P population, equation (4) agrees with his conclusion that the Orion Nebula is optically thick in Hα. While he finds some observational support for this view in an anomalous Hα/Hβ ratio, detection of the 3-cm line would provide direct confirmation of his findings.

Is it realistic to hope to detect such a small optical depth? Let us consider the transfer of radiation at λ 3 cm, taking into account the presence of continuum (free-free) emission as well as line emission. If we treat the nebula as a homogeneous sphere and at first treat only a radial ray, we may write the usual equation,

$$I_\nu = \frac{j_\nu}{\kappa_\nu}(1 - e^{-2\tau_\nu}) \simeq 2\,\frac{j_\nu}{\kappa_\nu}\,\tau_\nu(1-\tau_\nu) = 2\,j_\nu R(1-\tau_\nu)\,, \tag{5}$$

for the emergent intensity in the continuum. Here τ_ν is the continuum optical depth from the center to the surface; it is known to be <1 per cent at λ 3 cm in the Orion

Nebula. We may write a similar equation at the center of the line, including line emission and absorption. Taking the difference, we find (to the lowest order in τ_ν)

$$\Delta I_\nu \simeq 2R\left[j_\nu(l) - j_\nu(c)\,\tau_\nu(l)\right] = \tau_\nu(l)\left[2\,\frac{j_\nu(l)}{\kappa_\nu(l)} - I_\nu(c)\right]. \tag{6}$$

The first term is the spontaneous line emission; the second is absorption of the continuum by the line. Now

$$\frac{j_\nu(l)}{\kappa_\nu(l)} = \frac{2\,h\nu}{\lambda^2}\left(\frac{\varpi_P N_S}{\varpi_S N_P} - 1\right)^{-1} \simeq -\frac{2\,h\nu}{\lambda^2},$$

$$I_\nu(c) = \frac{2\,kT_B}{\lambda^2}, \tag{7}$$

$$\Delta I_\nu = \frac{2\,k\Delta T_B}{\lambda^2};$$

so equation (6) becomes

$$\Delta T_B = -\,\tau_\nu(l)\left(\frac{2\,h\nu}{k} + T_B\right). \tag{8}$$

Here T_B, the observed continuum brightness temperature, reaches 100° K (see below), while $h\nu/k$ is only 0 5° K. Therefore, the stimulated emission is one hundred times larger than the spontaneous emission, and we may neglect the latter. This result is quite different from Yada's; he ignored the stimulated emission, and his estimates of intensity are therefore far too pessimistic.

The line appears in emission due to negative absorption in the line as a result of the inverted level population. As the inverted population is due to "pumping" at another frequency (Lyman-α), we have here perhaps the first application of the "maser" principle in astronomy. Other examples of negative absorption have been discussed by Twiss (1958).

Equation (8) applies only to the central rays. Other rays will experience a smaller value of τ; without detailed knowledge of the structure of the nebula, we estimate a reduction by a factor of 2 in averaging ΔT_B over the face of the nebula. Hence we predict a change in antenna temperature of the nebula

$$\Delta T_A \simeq -\tfrac{1}{2}\,\tau T_A = +2\times 10^{-3} T_A\,. \tag{9}$$

Whether or not such a ΔT_A can be detected depends on T_A for the telescope in question, as well as on the radiometer sensitivity. Menon (1961) reports $T_A = 10°$ K for the Orion Nebula, using the Howard Tatel 85-foot telescope at λ 3.75 cm. As free-free emission is independent of wavelength, a similar figure should obtain at λ 3 cm, giving a predicted increment of antenna temperature

$$\Delta T_A = 0.02°\ \mathrm{K}\ \ (\text{Orion Nebula})\,. \tag{10}$$

Giordmaine (1960) has described a maser radiometer at 3-cm wave length which has an output fluctuation of 0.04° K. It appears that, with some improvements, the predicted emission line may soon be detectable.

Both T_A and sensitivity may improve considerably in the future. For example, if the 140-foot radio telescope under construction at the National Radio Astronomy Observatory performs according to expectations, with 60 per cent efficiency and a 3′ beam width, one could resolve the central position of the nebula where Menon considers that $T_B =$ 100° K, and T_A would then rise to perhaps 50° K and ΔT_A to 0.1° K. On the other hand, noise temperatures of maser radiometers may be considerably reduced from Giordmaine's value of 85° K. Giordmaine estimates that an order-of-magnitude reduction may be possible by better antenna design. Should such an improvement be realized, it would be found that the main noise contributor in the present experiment would be the Orion Nebula itself, with $T_A \simeq 50°$ K. Under these conditions the output fluctuations will be about

$$\Delta T = (Bt)^{-1/2} T_A \ , \tag{11}$$

and the ultimate sensitivity will be set by the bandwidth B and integrating time t. Here we are fortunate that the 3-cm line has an extremely large total half-power bandwidth of 100 Mc/s. If a maser of that bandwidth could be constructed and the integrating time set to 100 sec (as might be possible with a comparison radiometer suitable for detecting spectral lines), we would find the minimum detectable value of $\Delta T_A / T_A$ to be 10^{-5}. According to equation (9), this is two orders of magnitude smaller than the expected effect. We conclude that serious attempts to detect the 3-cm line would yield important information concerning the Lyman-α radiation field within diffuse nebulae.

We shall conclude with a brief discussion of the possibility of detecting the 3-cm line in the solar chromosphere. Dravskikh (1960) and Vitkevitch (1961) have indicated that experiments along these lines are already under way in the Soviet Union. A particularly favorable locale for Lymen-α pumping is the neighborhood of solar flares, where preliminary reports indicate that perhaps the effect has been detected (Vitkevitch 1961). Should the Lyman-α be intense enough, it may well be able to overcome the high 2P–2S collision rate and cause an emission line stimulated to emit by the background chromospheric brightness temperature of 10000° K. In this case the system noise will be due to the solar background, and the minimum detectable signal will be set by $(Bt)^{-1/2}$ alone, even with a rather noisy radiometer. Thus it may be hoped that even with present equipment an optical depth of 10^{-2}–10^{-3} could be detected, and such may very well exist. Care must, of course, be exercised to establish the true frequency dependence of any suspected event, to avoid confusion with other narrow-bandwidth solar phenomena.

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