1970

Can Discrete Sources Produce the Cosmic Microwave Radiation?

M. G. Smith

Bruce Partridge
Haverford College, bpartrid@haverford.edu

Repository Citation

(with M. G. Smith*) Can Discrete Sources Produce the Cosmic Microwave Radiation? Astrophysical Journal, Vol. 159, 737, March (1970).

1970

Can Discrete Sources Produce the Cosmic Microwave Radiation?

M. G. Smith

R. Bruce Partridge
Haverford College

Repository Citation

(with M. G. Smith*) Can Discrete Sources Produce the Cosmic Microwave Radiation? Astrophysical Journal, Vol. 159, 737, March (1970).

THE ASTROPHYSICAL JOURNAL, Vol. 159, March 1970

CAN DISCRETE SOURCES PRODUCE THE COSMIC MICROWAVE RADIATION?*

M. G. SMITH† AND R. B. PARTRIDGE
Palmer Physical Laboratory, Princeton University
Received 1969 June 16; revised 1969 September 12

ABSTRACT

After making a few general assumptions about the properties of discrete microwave sources, we calculate the amount of fluctuation the sources will produce in measurements of the temperature of the cosmic microwave background. By comparing these results with the available observations, we show that, if discrete sources produce the microwave background, then either the number of microwave sources per Mpc^3 must exceed the number of galaxies or the deceleration parameter must be less than 0.1.

The effect on the observable temperature fluctuations of scattering by an intergalactic plasma is taken into account.

I. INTRODUCTION

Since its discovery four years ago (Penzias and Wilson 1965), the cosmic background radiation in the microwave region of the spectrum has been variously interpreted. The original suggestion by Dicke *et al.* (1965) that the radiation is a relic of the hot initial phase of a "big bang" universe has been widely accepted. The current measurements (see Boynton, Stokes, and Wilkinson 1968; Bortolot, Clauser, and Thaddeus 1969; and references therein) of the spectrum of the radiation, with one possible exception (Shivanandan, Houck, and Harwit 1968), are in accord with this interpretation. On the other hand, a number of authors (Sciama 1966; Gold and Pacini 1968; Wolfe and Burbidge 1969) have suggested that the observed background might be the summed contributions of many discrete sources of microwave radiation.

The difficulties that such models have in conforming to the observed spectrum of the radiation have been discussed in detail by Wolfe and Burbidge. They conclude that if the parameters of the sources, such as spectral index and high-frequency cutoff, are chosen in the proper narrow range, such models can be made to fit the observations. They also discuss the number of sources which would be necessary to explain the intensity of the observed background.

To be valid, models of discrete sources must also be in agreement with observations of the isotropy of the cosmic background radiation. All of the spectral measurements and several of the early isotropy measurements of the background (e.g., Wilkinson and Partridge 1967) were made with instruments having very poor angular resolution. With such instruments, there was no hope of resolving discrete sources. In the past two years, several high-resolution studies of the background have been made (Conklin and Bracewell 1967*a*, *b;* Epstein 1967; Penzias, Schraml, and Wilson 1969). Adopting a few general assumptions, we shall show that the number of discrete sources must be very large to be compatible with the current measurements; otherwise, fluctuations in the background temperature would be larger than observed.

In deriving our results, we have taken into account possible scattering by intergalactic matter which will tend to reduce the observed fluctuations.

We consider here only evolutionary cosmological models. Penzias *et al.* (1969) and,

---

* This research was supported in part by the National Science Foundation and the Office of Naval Research of the U.S. Navy.

† Present address: MITRE Corporation, Bedford, Massachusetts.

in a very recent paper, Hazard and Salpeter (1969) have shown that the narrow range of steady-state models which reproduce the spectrum of the microwave background are not consistent with the available measurements of the small-scale isotropy of the background.

II. ASSUMPTIONS

We list here a series of general assumptions about the nature and evolution of discrete sources. No attempt is made to develop a detailed model for the sources, such as that of Wolfe and Burbidge (1969). Instead, we prefer to make a minimum number of general assumptions and, from these, to derive a general relation between the number density of sources and the amplitude of fluctuations in the background temperature. Several of the assumptions are conservative in the sense that a relaxation of the assumptions would *increase* the amplitude of the fluctuations.

1. *All sources have the same apparent luminosity,* $\mathfrak{L}_0$, *in the absence of intergalactic extinction.* That is, if there were no intergalactic scattering, all sources would appear equally luminous, independent of their distance from the observer. We recognize that this is not a very realistic assumption: in general, nearby sources will have a greater apparent luminosity. In this case, however, the assumption is conservative: if a few nearby sources were to contribute a larger fraction of the radiation received, the fluctuations in the observed antenna temperature would be larger. (Likewise, if there were a range in the intrinsic luminosities of the sources, larger fluctuations would result.)

Only if more distant sources have *greater* apparent luminosities than nearby ones will the fluctuations be decreased. This sort of breakdown of assumption 1 may occur if:

a) The sources were much more luminous in the past. Such an evolution is incompatible with a steady-state cosmology. For evolutionary cosmologies, a luminosity dependence stronger than $(1 + z)^m$, with $m \gtrsim 2$, is needed to violate our assumption 1.[1]

b) The source spectrum increases sharply with frequency throughout the centimeter and millimeter regions. A frequency dependence stronger than ν^2 is required to violate assumption 1.[2]

2. *The sources are statistically independent.* First, it is evident that a tendency for the sources to cluster would cause an increase in the small-scale anisotropy of the radiation. Also implicit in this assumption is that the sources are either point sources or optically thin, so that they will not obscure each other if they overlap.

We suggest that it is unlikely that the sources will be large and at the same time optically thick. Many possible microwave sources such as QSOs and Seyfert nuclei have small proper diameters, and may be treated essentially as point sources. Other conceivable sources, such as spiral galaxies, are optically thin at centimeter and millimeter wavelengths.

In the unlikely event that the sources *are* optically thick and *do* begin to obscure one another at a redshift less than $z(t = 10^8$ years), it will be true that extinction caused by foreground sources will tend to reduce observed fluctuations in the background. This follows from the fact that the scattering or absorption and reemission processes will, in general, be isotropic. In this sense, optically thick sources act in the same way as an intergalactic plasma. In the following section, we show that Thomson scattering by an intergalactic plasma does not strongly affect our results in the region of greatest interest, where the variable μ is smallest (compare Figs. 2 and 3). There are two arguments which indicate that extinction by optically thick sources will have an even weaker effect. First,

---

[1] The actual value of m depends on the cosmological model considered. Note that all models of Wolfe and Burbidge (1969) take $m = -\frac{1}{2}$.

[2] The model spectra assumed by Wolfe and Burbidge (1969) have strong frequency dependences. However, because of their assumption about the evolution of the sources, the majority of their models fit our assumption 1. Direct calculations show that nearby sources are brighter than more distant ones for the following models of Wolfe and Burbidge: all models with their parameter $\beta = 2.2$; models with $\nu_0/\nu_c \lesssim 50$, $\beta = 2.5$; and models with $\nu_0/\nu_c \lesssim 10$, $\beta = 3.0$.

the total matter density of the sources must necessarily be less than the total density of metagalactic matter for a given cosmological model. In addition, many absorption and scattering processes that might play a role in making the sources optically thick are less efficient than Thomson scattering. That is, the total extinction cross-section per gram of material is less than for a completely ionized plasma of hydrogen. This is true, for example, for the extinction of millimeter and centimeter radiation by small particles. Taken together, these arguments indicate that even if the sources do obscure each other, the effect on our conclusions will be no larger than the effect produced by an intergalactic plasma.

In view of these arguments, we consider the assumption of statistical independence reasonably well established, and will proceed on that basis.

3. *The sources appear only for epochs greater than 10^8 years.* We assume implicitly that the sources are related to galaxies or to objects of roughly galactic size and density. From Partridge and Peebles's (1967) work on galaxy formation, 10^8 years appears to be a lower limit on the epoch of formation of galaxies or other objects with a similar mean density.

In the calculations that follow, we shall actually work with a variable z' for the redshift at which the discrete sources first appear. Thus our assumption is equivalent to

$$z' \leq z(t = 10^8 \text{ years}) .$$

The functional dependence of z' on t is taken from Landau and Lifshitz (1962). We assume here and throughout standard homogeneous cosmological models in which the cosmological constant $\Lambda = 0$.

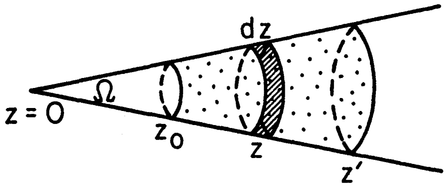

FIG. 1.—Schematic representation of the model discussed in the text; dots represent discrete sources

This assumption is not conservative: that is, if the sources form before $t = 10^8$ years, the expected fluctuation level is somewhat less (see Figs. 2 and 3 for large values of z').

For some calculations, we will make an additional assumption:

4. *The sources are not visible for redshifts less than some* z_0; that is, they "turn off" at some time in the past. This assumption allows us to exclude the few very nearby sources which would tend to increase the fluctuation in the background. For most cosmological models, it is again conservative. The effect of relaxing the assumption is shown in Figure 4: if $z_0 \rightarrow 0$, an even larger number of sources is needed to conform with the observations of small-scale isotropy.

III. THE RELATION BETWEEN NUMBER DENSITY AND FLUCTUATION

Consider the situation shown schematically in Figure 1. Here Ω is the solid angle of a radiometer beam. Under our assumptions, sources exist between redshifts z_0 and z'. There are $dn(z)$ sources in the beam between z and $(z + dz)$. With the simple assumptions that all sources have the same apparent luminosity and are statistically independent, the differential variance in the luminosity due to sources in dz is proportional to the number of sources:

$$d\sigma_I^2 = \mathfrak{L}^2(z)dn(z) ,$$

where $\mathfrak{L}(z)$ is the apparent luminosity of a single source at a redshift z.

We must now consider the effect of scattering by intergalactic matter. Let $\tau(z)$ be the optical depth at a redshift z. Then the intensity of radiation received from a single source at z is

$$\mathfrak{L}(z) = \mathfrak{L}_0 e^{-\tau(z)} \, .$$

Here, $\mathfrak{L}_0$ is the apparent luminosity of a source where $\tau = 0$. By assumption (2), the total variance σ_I^2 is the integral over z of the variance produced by each segment dz, i.e.,

$$\sigma_I^2 = \int_{z_0}^{z'} d\sigma_I^2 = \mathfrak{L}_0^2 \int_{z_0}^{z'} e^{-2\tau(z)} dn(z) \, . \tag{1}$$

Since the radiation is scattered and not absorbed,

$$I = \mathfrak{L}_0 \int_{z_0}^{z'} dn(z) = \mathfrak{L}_0 N \, , \tag{2}$$

where N is the total number of sources in the beam between z' and z_0. From equations (1) and (2), the small-scale anisotropy is

$$\frac{\sigma_I}{I} = \frac{1}{N} \left[\int_{z_0}^{z'} e^{-2\tau(z)} dn(z) \right]^{1/2} . \tag{3}$$

The only significant, and most efficient, cause of scattering in the intergalactic medium is Thomson scattering by free electrons in the intergalactic plasma (Smith 1969). Using the results of Gunn and Peterson (1965), we obtain

$$\tau(z) = \frac{c n_0 \sigma_{\mathrm{T}}}{H_0} \left[\frac{3q_0 + q_0 z - 1}{3q_0^2} (1 + 2q_0 z)^{1/2} - \frac{3q_0 - 1}{3q_0^2} \right] , \tag{4}$$

where σ_{T} is the Thomson-scattering cross-section; H_0 is the present value of the Hubble constant, taken as 100 km sec^{-1} Mpc^{-1}; q_0 is the deceleration parameter; and n_0 is the present number density of free electrons. To be conservative, we find n_0 as follows. Let the matter density for a particular cosmological model be ρ, and let ρ_g be the Oort limit for the density of matter in visible galaxies, about 5×10^{-31} g cm^{-3} (see Peebles and Partridge 1967). Then we assume that all the "missing matter" $(\rho - \rho_g)$ is in the form of ionized hydrogen, so $n_0 = (\rho - \rho_g)/m_p$, where m_p is the proton mass. Other plausible forms of intergalactic matter, such as neutral hydrogen, ionized helium, or condensed matter, will produce less scattering per gram of material.

It remains to evaluate N. We employ the standard metric (McVittie 1965)

$$ds^2 = dt^2 - \frac{R^2(t)}{c^2} [d\omega^2 + \phi_K^2(\omega)(d\theta^2 + \sin^2 \theta d\phi^2)]$$

which yields the differential proper volume

$$dV = R^3(t) \phi_K^2(\omega) d\omega \Omega \, ,$$

where we again take Ω as the solid angle of the radiometer beam. Let $\nu(z)$ be the number density of sources at a redshift z. Then

$$dn(z) = \nu(z) R^3(t) \phi_K^2(\omega) d\omega(z) \Omega \, . \tag{5}$$

Now, $R(t(z)) = R_0(1 + z)^{-1}$ and $\nu(z) = \nu_0(1 + z)^3$, where R_0 and ν_0 are the values of R and ν at the present epoch, so

$$dn(z) = \nu_0 \Omega R_0^3 \phi_K^2[\omega(z)] d\omega(z) \, . \tag{6}$$

Finally $\omega(z)$ may be found by integrating

$$d\omega(z) = \frac{cH_0^{-1}dz}{R(t(z))(1+z)^2(1+2q_0z)^{1/2}} \tag{7}$$

(from Gunn and Peterson 1965).

Combining equations (3) and (6), we have

$$\nu_0 = \left[\Omega\left(\frac{\sigma_I}{I}\right)^2 R_0^3\right]^{-1}\left\{\int_{z_0}^{z'} e^{-2\tau(z)}\phi_K^2[\omega(z)]d\omega(z)\right\}\left\{\int_{z_0}^{z'} \phi_K\,[\omega(z)]d\omega(z)\right\}^{-2} \tag{8}$$

for the number density of sources which will produce a fractional fluctuation σ_I/I in measurements made by a radiometer with a beam of solid angle Ω. Clearly, for measurements of antenna temperature (a quantity proportional to power),

$$\sigma_T/T = \sigma_I/I\,,$$

where σ_T is the fluctuation in the antenna temperature.

To keep the results general, it is convenient to work with the quantity $\mu = \nu_0\Omega(\sigma_T/T)^2$. Using equations (4) and (7), we integrated expression (8) numerically for various values of z', z_0, and q_0. The results allow us to plot μ as a function of z', as in Figure 2.

With assumption (3) above, portions of the curves to the right of the dashed line in Figure 2 are excluded. Thus the extreme case is for models with $q_0 \sim 0.02$, corresponding to a density of $\sim 8 \times 10^{-31}$ g cm^{-3}, only 50 percent above the Oort limit (for which $q_0 = 0.0136$). Models with higher values of q_0 require larger number of sources. We see that, even with an intergalactic plasma which tends to reduce the fluctuations, $\mu \gtrsim 3 \times 10^{-6}$ square minutes of arc per Mpc^3.

We have also evaluated equation (8) for the simple case in which there is no intergalactic matter, so that $\tau(z) = 0$ for all z (see Fig. 3). In this case μ is a minimum for $q_0 = 0.0136$, the lowest-density cosmological model consistent with the Oort limit for mass in visible galaxies.

Both Figure 2 and Figure 3 show results with z_0 taken as zero, the case in which the sources remain visible until the present. The effect on μ of "turning off" the sources at some time in the past is demonstrated in Figure 4: for most values of q_0, indicated by the vertical bars, raising z_0 decreases μ. For the lowest values of q_0, changing z_0 has only a small effect on μ. In this figure, the solid lines are curves like the dashed lines in Figures 2 and 3, representing the locus $z'(t = 10^8$ years) for models with different values of q_0.

IV. COMPARISON WITH OBSERVATIONS, AND CONCLUSIONS

Can currently available observations help to rule out models of discrete sources? The most sensitive measurement to date appears to be that of Conklin and Bracewell (1967*b*), which was made at a wavelength of 3 cm. On a scale of ~12′, they set an upper limit of 0.0036° K on temperature fluctuations in the background. (These are revised values, which differ somewhat from the published results [E. K. Conklin, private communication]. Also, to obtain this value, the internal receiver noise has been subtracted from the observed fluctuation level.) However, Conklin and Bracewell did not allow for the fact that not all the received power was in the main 12′ lobe of the antenna. Conklin (private communication) estimates that this effect reduces the sensitivity of the measurement by a factor of 0.75. With this correction, $\sigma_T = 0.0048°$ K. Taking 2.7° K as the thermodynamic temperature of the background radiation (Stokes, Partridge, and Wilkinson 1967), we have

$$\left(\frac{\sigma_T}{T}\right)^2\Omega = 4 \times 10^{-4} \text{ square minutes of arc.}$$

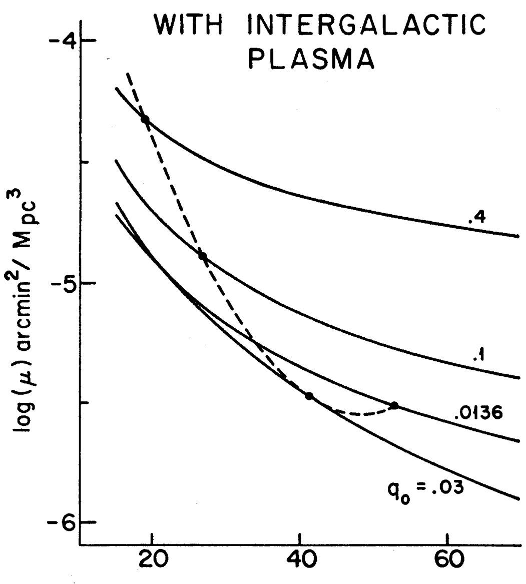

FIG. 2

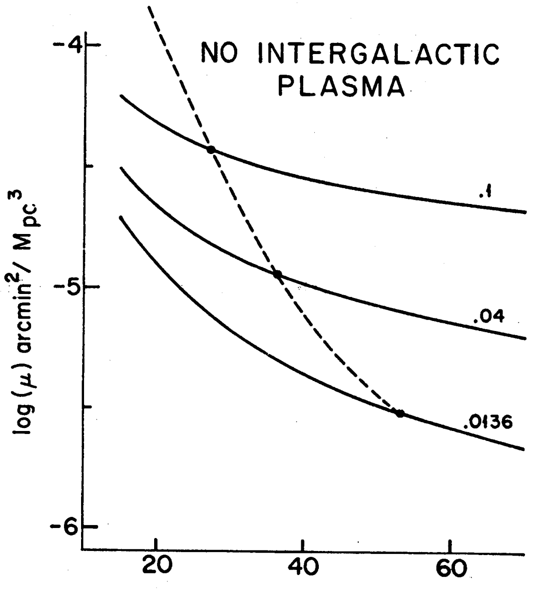

FIG. 3

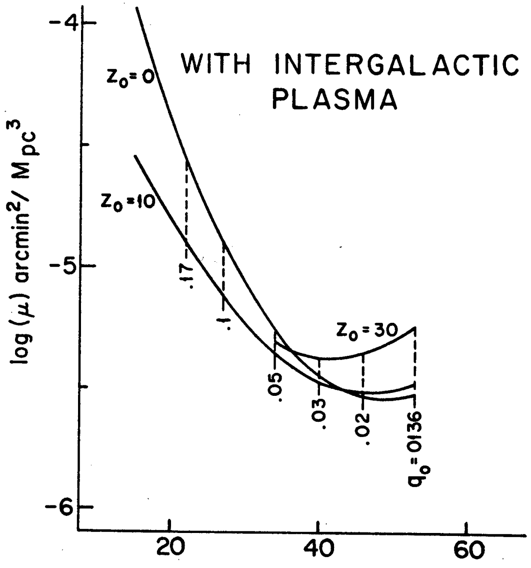

FIG. 4

FIG. 2.—Solid lines show the variable μ as a function of z', the redshift at which discrete sources turn on. Here, z_0 is taken as zero. Results for several values of the deceleration parameter are shown. The curve for $q_0 = 0.03$ lies below the curve for $q_0 = 0.0136$ (corresponding to $\rho = \rho_a$) because of the reduction in fluctuation produced by intergalactic scattering. Dashed line shows the cutoff imposed by assumption 3 ($t > 10^8$ yr): regions to the right of the dashed curve are excluded. The minimum allowable value of μ is thus 2.8×10^{-6}.

FIG. 3.—Resembles Fig. 2, except that no intergalactic scattering is assumed. Solid curves are labeled by the appropriate values of q_0: note that μ increases rapidly for larger q_0.

FIG. 4.—Here we choose to plot the relation between μ and z' in a somewhat different form, where z_0 is the redshift at which sources turn off. Each solid curve represents the locus of points where $t = 10^8$ yr for a range of values of q_0. (Compare $z_0 = 0$ curve here to dashed curve in Fig. 2.) The values of q_0 are indicated by vertical dashes along the curves. Regions below the solid curves and to the right of the vertical dashed lines are excluded for each set of values of z_0 and q_0. For example, the *minimum* value for μ with $z_0 = 10$ and $q_0 = 0.1$ is $\sim 8 \times 10^{-6}$. Note that μ is generally highest for $z_0 = 0$. Thus, if the sources remain luminous until the present, their number density must be higher to fit the observations.

Penzias, Schraml, and Wilson (1969) have recently reported a measurement made at 3.5 mm. Their value[3] for σ_T is 0.04° K, and for Ω, 0.0014 square degrees, or 5.0 square minutes of arc. However, at 3.5 mm the antenna temperature corresponding to a thermodynamic temperature of 2.7° K is only 1.15° K. Hence $(\sigma_T/T)^2\Omega = 6 \times 10^{-3}$ square minutes of arc.

Epstein's (1967) results, also at ~3 mm, are an order of magnitude less sensitive than those of Penzias *et al.*

To compare these results with the plots of μ in Figures 2, 3 and 4, we take $\nu_0 = 0.03$ Mpc^{-3}, the present number density of galaxies (Allen 1963). With this value, the results of Conklin and Bracewell establish an upper limit $\mu \leq 1.2 \times 10^{-5}$. Thus models of discrete sources with $q_0 > 0.1$ are ruled out, even if intergalactic plasma exists. If there is no intergalactic plasma, the implied upper limit on q_0 is lower, ~0.05. These results place strong constraints on any model of discrete sources for the background which conforms to the general assumptions made above.

We recognize that the number of galaxies may well exceed 0.03 Mpc^{-3}. On the other hand, it is reasonable to suppose that only a fraction of all galaxies will be microwave sources. In this connection, it is worth noting that Seyfert galaxies, which have been suggested as possible sources for intense microwave radiation, make up ~1 percent of the total number of galaxies (Burbidge, Burbidge, and Sandage 1963). Thus Seyfert galaxies can contribute only a small fraction of the observed microwave flux at 3 cm. The same argument applies a fortiori to QSOs provided that they are at cosmological distances.

Finally, if ν_0 is reduced to 0.007 Mpc^{-3}, which is about 25 percent of the number density of galaxies, the observations of Conklin and Bracewell rule out *all* discrete source models which fit our general assumptions.

We would like to thank Drs. Paul E. Boynton, E. K. Conklin, and A. A. Penzias for useful discussions, and an anonymous referee for several pertinent and helpful comments.

REFERENCES

Allen, C. W. 1963, *Astrophysical Quantities* (London: The Athlone Press).
Bortolot, V. J., Clauser, J. F., and Thaddeus, P. 1969, *Phys. Rev. Letters*, **22**, 307.
Boynton, P. E., Stokes, R. A., and Wilkinson, D. T. 1968, *Phys. Rev. Letters*, **21**, 462.
Burbidge, G. R., Burbidge, E. M., and Sandage, A. R. 1963, *Rev. Mod. Phys.*, **35**, 947.
Conklin, E. K., and Bracewell, R. N. 1967*a*, *Phys. Rev. Letters*, **18**, 614.
———. 1967*b*, *Nature*, **216**, 777.
Dicke, R. H., Peebles, P. J. E., Roll, P. G., and Wilkinson, D. T. 1965, *Ap. J.*, **142**, 414.
Epstein, E. E. 1967, *Ap. J.* (*Letters*), **148**, L157.
Gold, T., and Pacini, F. 1968, *Ap. J.* (*Letters*), **152**, L115.
Gunn, J. E., and Peterson, B. A. 1965, *Ap. J.*, **142**, 1633.
Hazard, C., and Salpeter, E. E. 1969, *Ap. J.* (*Letters*), **157**, L87.
Landau, L. D., and Lifshitz, E. M. 1962, *The Classical Theory of Fields* (2d ed.; Reading, Mass.: Addison-Wesley Publishing Co.).
McVittie, G. C. 1965, *General Relativity and Cosmology* (2d ed.; London: Chapman and Hall).
Partridge, R. B., and Peebles, P. J. E. 1967, *Ap. J.*, **147**, 868.
Peebles, P. J. E., and Partridge, R. B. 1967, *Ap. J.*, **148**, 713.
Penzias, A. A., and Wilson, R. W. 1965, *Ap. J.*, **142**, 419.
Penzias, A. A., Schraml, J., and Wilson, R. W. 1969, *Ap. J.* (*Letters*), **157**, L49.
Sciama, D. W. 1966, *Nature*, **211**, 277.
Shivanandan, K., Houck, J. R., and Harwit, M. O. 1968, *Phys. Rev. Letters*, **21**, 1460.
Smith, M. G. 1969, unpublished senior thesis, Princeton University.
Stokes, R. A., Partridge, R. B., and Wilkinson, D. T. 1967, *Phys. Rev. Letters*, **19**, 1199.
Wilkinson, D. T., and Partridge, R. B. 1967, *Nature*, **215**, 719.
Wolfe, A. M., and Burbidge, G. R. 1969, *Ap. J.*, **156**, 345.

---

[3] We have adjusted their quoted value to take account of the fact that not all the received power was in the main lobe of their antenna (see Penzias *et al.* 1969).