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Geometric Tests of Cosmological Models: I. Probing Dark Energy Using the Kinematics of High Redshift Galaxies,

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Geometrical tests of cosmological models

I. Probing dark energy using the kinematics of high redshift galaxies

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ABSTRACT

We suggest to use the observationally measured and theoretically justified correlation between size and rotational velocity of galactic discs as a viable method to select a set of high redshift standard rods which may be used to explore the dark energy content of the universe via the classical angular-diameter test. Here we explore a new strategy for an optimal implementation of this test. We propose to use the rotation speed of high redshift galaxies as a standard size indicator and show how high resolution multi-object spectroscopy and ACS/HST high quality spatial images, may be combined to measure the amplitude of the dark energy density parameter Ω_Q, or to constrain the cosmic equation of state parameter for a smooth dark energy component ($w = p/\rho$, $-1 \leq w < -1/3$). Nearly 1300 standard rods with high velocity rotation in the bin $V = 200 \pm 20$ km s^{-1} are expected in a field of 1 sq. degree and over the redshift baseline $0 < z < 1.4$. This sample is sufficient to constrain the cosmic equation of state parameter w at a level of 20% (without priors in the [Ω_m, Ω_Q] plane) even when the [OII]λ3727 Å linewidth-diameter relationship is calibrated with a scatter of ~40%. We evaluate how systematics may affect the proposed tests, and find that a linear standard rod evolution, causing galaxy dimensions to be up to 30% smaller at $z = 1.5$, can be uniquely diagnosed, and will minimally bias the confidence level contours in the [Ω_Q, w] plane. Finally, we show how to derive, without a priori knowing the specific functional form of disc evolution, a cosmology-evolution diagram with which it is possible to establish a mapping between different cosmological models and the amount of galaxy disc/luminosity evolution expected at a given redshift.

Key words. cosmology: observations – cosmology: theory – cosmology: cosmological parameters – galaxies: high-redshift – galaxies: fundamental parameters – galaxies: evolution

1. Introduction

Several and remarkable progresses in the understanding of the dynamical status of the universe, encourage us to believe that, after roaming from paradigm to paradigm, we are finally converging towards a well-founded, internally consistent standard model of the universe.

The picture emerging from independent observations and analysis is sufficiently coherent to be referred to as the *concordance* model (e.g. Tegmark 2006). Within this framework, the universe is flat ($\Omega_K = -0.003^{+0.0095}_{-0.0102}$) composed of ~1/5 cold dark matter ($\Omega_{cdm} \sim 0.197^{+0.016}_{-0.015}$) and ~3/4 dark energy ($\Omega_\Lambda = 0.761^{+0.017}_{-0.018}$), with large negative pressure ($w = -0.941^{+0.017}_{-0.018}$), and with a very low baryon content ($\Omega_b = 0.0416^{+0.0019}_{-0.0018}$). Mounting and compelling evidence for accelerated expansion of the universe, driven by a dark energy component, presently relies on our comprehension of the mechanisms with which Supernovae Ia (SNIa) emit radiation (see Perlmutter et al. 1999; Riess et al. 2001) and of the physical processes that produced temperature fluctuations in the primeval plasma (see

Lee et al. 2001; de Bernardis et al. 2002; Halverson et al. 2002; Spergel et al. 2006).

Even if the ambitious task of determining geometry and evolution of the universe as a whole, which commenced in the 1930s, now-day shows that the relativistic Friedman-Lemaître model passes impressively demanding checks, we are faced with the challenge of developing and adding new lines of evidence supporting (or falsifying) the *concordance* model. Moreover, even if we parameterize our ignorance about dark energy describing its nature only via a simple equation of state $w = p/\rho$, we only have loose constraints on the precise value of the w parameter or on its functional behavior.

In this spirit we focus this analysis on possible complementary approaches to determining fundamental cosmological parameters, specifically on geometrical tests.

A whole arsenal of classical geometrical methods has been developed to measure global properties of the universe. The central feature of all these tests is the attempt to directly probe the various operative definitions of relativistic distances by means of scaling relationships in which an observable is expressed as a function of redshift (z) and of the fraction of critical density contributed by all forms of matter and energy (Ω).

The most remarkable among these classical methods are the *Hubble diagram* (or magnitude-redshift relation $m = m(M, z, \Omega)$), the *Angular diameter test* (or angle-redshift

relation $\theta = \theta(L, z, \Omega)$), the *Hubble test* (or count-redshift relation $N = N(n, z, \Omega)$) or the *Alcock-Paczinsky test* (or deformation-redshift relation $\Delta z / z \Delta \theta \equiv k = k(z, \Omega)$). The common key idea is to constrain cosmological parameters by measuring, at various cosmic epochs, the scaling of the apparent values m, θ, N, k of some reference standard in luminosity (M), size (L), density (n) or sphericity and compare them to corresponding model predictions.

The observational viability of these theoretical strategies has been remarkably proved by the Supernova Cosmology Project (Perlmutter et al. 1999) and the High-z Supernova Team (Riess et al. 2001) in the case of the Hubble diagram. With a parallel strategy, Newman et al. (2002) recently showed that a variant of the Hubble test ($N(z)$ test) can be in principle applied to distant optical clusters selected in deep redshift survey such as VVDS (Le Fèvre et al. 2005) and DEEP2 (Davis et al. 2000), in order to measure the cosmic equation-of-state parameter w.

Unfortunately, the conceptually simple pure geometrical tests of world models, devised to anchor relativistic cosmology to an observational basis, have so far proved to be difficult to implement. This is because the most effective way to constrain the evolution of the cosmological metric consists in probing deep regions of the universe with a primordial class of cosmological objects. Besides the complex instrumental technology this kind of experiments requires, it becomes difficult at high redshift to disentangle the effects of object evolution from the signature of geometric evolution.

Since geometrical tests are by definition independent from predictions of theoretical models or simulations, as well as from assumptions about content, quality and distribution of matter in the universe (mass fluctuations statistics, e.g. Haiman et al. 2001; Newman et al. 2001, galactic biasing, e.g. Marinoni et al. 1998; Lahav et al. 2001; Marinoni et al. 2006, Halo occupation models, e.g. Berlind et al. 2001; Marinoni & Hudson 2002; van den Bosch et al. 2006) it is of paramount importance to try to devise an observational way to implement them. The technical maturity of the new generation of large telescopes, multi object spectrographs, large imaging detectors and space based astronomical observatories will allow these tests to be more effectively applied in the near future (Huterer & Turner 2000). In this paper, we describe a method to select a class of homologous galaxies that are at the same time standard in luminosity and size, that can be in principle applied to data coming from the zCOSMOS spectro-photometric survey (Lilly et al. 2006); of the deep universe.

An observable relationship exists between the speed of rotation V of a spiral galaxy and its metric radial dimension D as well as its total luminosity L (Tully & Fisher 1977; Bottinelli et al. 1980). From a theoretical perspective, this set of scaling relations are expected and explicitly predicted in the context of CDM models of galaxy formation (Mo et al. 1998). The Tully-Fisher relations for diameter and luminosity have been extensively used in the local universe to determine the distances to galaxies and the value of the Hubble constant. We here suggest that they may be used in a cosmological context to select in a physically justified way, high redshift standard rods since galaxies having the same rotational speed will statistically have the same narrow distribution in physical sizes.

The picture gets complicated by the fact that the standard model of the universe implies some sort of evolution in its constituents. In a non static, expanding universe, where the scale factor changes with time, we expect various galaxy properties, such as galaxy metric dimensions, to be an explicit function of redshift. In principle, one may break this circular argument

between model and evolution with two strategies: either by understanding the effects of different standard rod evolutionary patterns on cosmological parameters, or by looking for cosmological predictions that are independent from the specific form of the disc evolution function.

In this paper, following the first approach, we study how different disc evolution functions may bias the angular-diameter test. We simulate the diameter-redshift experiment using the amount of data and the realistic errors expected in the context of the zCOSMOS survey. We then evaluate how different disc evolution functions may be unambiguously recognized from the data and to what extent they affect the estimated values of the various cosmological parameters. We also explore the second approach and show how cosmological information may be extracted, without any knowledge about the particular functional form of the standard rod evolution, only by requiring as a prior an estimate of the upper limit value for the relative disc evolution at some reference redshift.

This paper is set out as follow: in Sect. 2 we review the theoretical basis of the angular diameter test. In Sect. 3 we describe the proposed strategy to select high redshift standard rods. In Sect. 4 we digress on how implement in practice the "$\theta - z$" test with zCOSMOS data, and in Sect. 5 we present the zCOSMOS expected statistical constraints on cosmological parameters. In Sect. 6 we discuss different possible approaches with which to address the problem of standard rod evolution. Conclusions are drawn in Sect. 7.

2. The angular diameter test

We investigate the possibility of probing the cosmological metric using the redshift dependence of the apparent angular diameter of a cosmic standard rod. What gives this test special appeal is the possibility of detecting the "*cosmological lensing*" effect, which causes incremental magnification of the apparent diameter of a fixed reference length.

Let's consider the transverse comoving distance (see Hogg 1999)

$$r(z, \Omega_m, \Omega_Q, w) = \frac{c}{H_0 \sqrt{|\Omega_k|}} S_k \left(\sqrt{|\Omega_k|} \int_0^z E(x)^{-1} \mathrm{d}x \right) \quad (1)$$

where

$$E(x) = \left[\Omega_m (1+x)^3 + \Omega_Q (1+x)^{3+3w} + \Omega_k (1+x)^2 \right]^{1/2} \quad (2)$$

and where $S_{-1}(y) = \sinh(y)$, $S_1(y) = \sin(y)$, $S_0(y) = y$ while $\Omega_k = 1 - \Omega_m - \Omega_Q$.

An object with linear dimension D at a redshift z has thus an observed angular diameter θ

$$\theta(z, \boldsymbol{p}) = \frac{D}{r(z, \boldsymbol{p})} (1+z) \quad (3)$$

which depends on the general set of cosmological parameters $\boldsymbol{p} = [\Omega_m, \Omega_Q, w]$ via the relativistic definition of angular distance, $d_A = r(z, \boldsymbol{p})/(1+z)$.

This test may be implemented without requiring the knowledge of the present expansion rate of the universe (the dependence from the Hubble constant cancels out in Eq. (3)). At variance, although characterized by a smooth and diffuse nature, dark energy significantly affect the dynamic of the universe. From Eq. (1) it is clear that the angular-diameter test depends on the dark energy component via the expansion rate evolution $E(z)$.
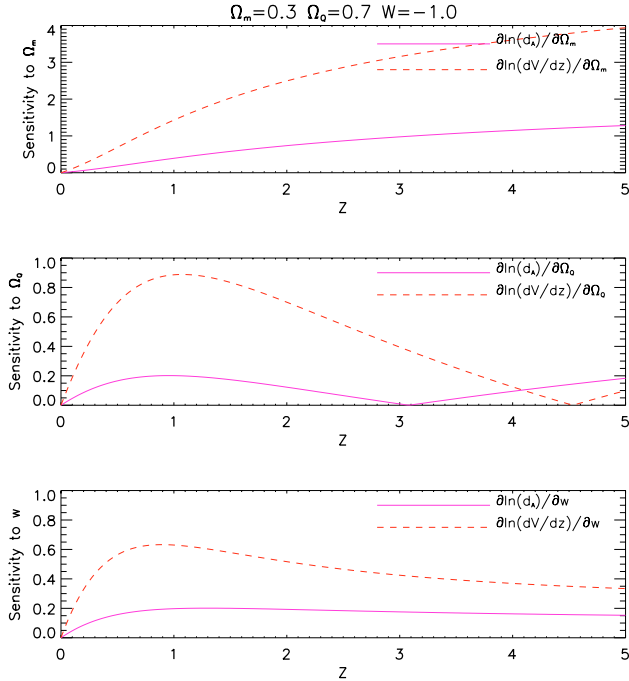
Fig. 1. The relative sensitivity of the angular diameter distance (d_A) and volume element (dV/dz) to a change in the values of Ω_m, Ω_Q and w. The partial derivatives are computed with respect to the position ($\Omega_m = 0.3, \Omega_Q = 0.7, w = -1$) in the parameter space.

The more negative w, the more accelerated the expansion is and the smaller a fixed standard rod will appear to an observer.

The efficiency of different cosmological observables in probing the nature of space-time ultimately depends upon their sensitivity to the cosmological parameters Ω_m, Ω_Q, w. The relative sensitivity of empirical cosmological tests based on the scaling of the angular diameter distance (d_A) and of the volume element (dV/d$z = (c/H_0)(r^2/E(z))$) is derived in Fig. 1, where we assume that Poissonian errors are constant in time and no redshift-dependent systematics perturb the measurements (e.g. Huterer & Turner 2000). Since the luminosity distance (i.e. the distance inferred from measurements of the apparent magnitude of an object of known absolute luminosity) is defined as $d_L = (1 + z)^2 d_A$, we note that the angular diameter test has the same cosmological discriminatory power as the Hubble diagram. The upper panel of Fig. 1 shows that the sensitivity of both d_A and dV/dz to the mean mass density parameter, Ω_m, increases monotonically as a function of redshift. This means that the deeper the region of the universe surveyed, the more constrained the inferred value of Ω_m is.

Conversely, the sensitivity of both empirical tests to a change in the constant value of w peaks at redshift around unity, and levels off at redshifts greater than ~5. The reason for this is that the dark energy density ρ_Q, which substantially contributes to the present-day value of the expansion rate was negligible in the early universe ($\rho_Q/\rho_M \propto (1 + z)^{3w}$, see Eq. (2)).

The fact that we are living in a special epoch, when two or more terms in the expansion rate equation make comparable contributions to the present value of $E(z)$, can be appreciated in the central panel of Fig. 1. Because each of the terms in Eq. (2) varies with cosmic time in a different way, there is a redshift window where the search for Ω_Q is less efficient (i.e. $2 < z < 4$). Therefore one can maximize the cosmological information which can be extracted from the classical tests of

cosmology, and specifically from the angular diameter test, by devising observational programs probing a large field of view in the redshift range $0 \leq z \leq 2$.

3. The standard rod

A variety of standard rod candidates have been explored in previous attempts of implementing the angular diameter-redshift test: galaxies (Sandage 1972; Djorgovski & Spinrad 1981), clusters (Hickson 1977; Bruzual & Spinrad 1978; Pen 1997), halo clustering (Cooray et al. 2001). Those methods failed to yield conclusive evidence because the available redshifts were few and local, and the quality of the imaging data used in the estimate of sizes was poor.

Good quality size measurements for high redshift objects have become available for radio sources (e.g. Miley 1971; Kapahi 1975; and recently several authors Kellermann 1993; Wilkinson et al. 1998, have reported a redshift dependence of radio source angular sizes at $0.5 < z < 3$, which is not easily reconciled with other recent measurements of the cosmological parameters (but see Daly & Djorgovski 2004, for results more consistent with the concordance model.)

The radio source results may be affected by a variety of selection and evolutionary effects, the lack of a robust definition of size, and by difficulties in assembling a large, homogeneous sample of radio observations (Buchalter et al. 1998; Gurvits et al. 1999).

A common thread of weakness in all these studies is that there are no clear criteria by which galaxies, clusters, extended radio lobes or compact radio jets associated with quasars and AGNs should be considered universal standard rods. Moreover, lacking any local calibration for the metric size of the standard rod, the standard rod dimension (parameter D in Eq. (3)) is often considered as a free fitting parameter. Since the inferred cosmological parameters heavily depend on the assumed value for the object size (Lima & Alcaniz 2002), an a priori independent statistical study of the standard rod distribution properties is an imperative prerequisite.

We thus propose to use information on the kinematics of galaxies, as encoded in their optical spectrum, a) to identify in an objective and empirically justified way a class of objects behaving as standard rods, and b) to measure the absolute value of the standard rod length. The basic idea consists in using the velocity-diameter relationship for disc galaxies (e.g. Tully & Fisher 1977; Saintonge et al. 2008) as a cosmological metric probe. In Fig. 2 we plot the local relationship we have derived in Paper II of this series (Saintonge et al. 2008) between half-light diameters and rotational velocities inferred using the $H_\alpha \lambda 6563$ Å line. The sample used to calibrate the diameter-velocity relationship is the SFI++ sample described by Springob et al. (2007). Also shown is the the amplitude of the scatter in the zero-point calibration of the standard rods.

In the local universe, rotation velocities can be estimated from either 21 cm HI spectra or from the $H_\alpha \lambda 6563$ Å optical emission lines. However, the H_α line is quickly redshifted into the near-infrared and cannot be used in ground-based optical galaxy redshift surveys at $z > 0.4$, while HI is not detectable much past $z = 0.15$. Only [OII]$\lambda 3727$ Å line widths can be successfully used in optical surveys to infer the length of a standard rod dimension D at $z \sim 1$. Clearly one could obtain rotational velocity information for high redshift objets by observing the H_α line with near-IR spectrographs. However it is much easier to get large samples of kinematic measurements using OII and
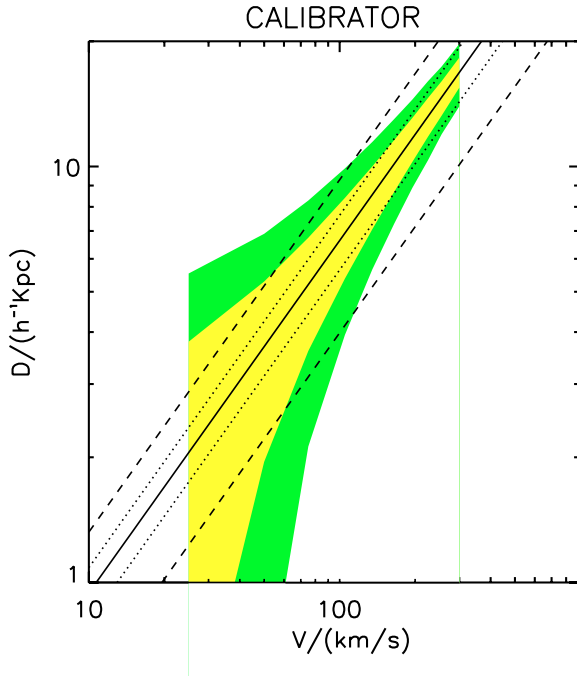
Fig. 2. The diameter vs. velocity relationship calibrated in Paper II is plotted using a black line. Shadowed regions represent the 1 and 2σ uncertainties in the calibrated relationship. D represents the corrected (face on) half light diameter while the velocity has been measured using the H$_\alpha$ line. Dotted lines represent the upper and lower relative uncertainty $[\sigma_D/D]_{int} = 0.15$ in diameters. The *conservative* relative dispersion in the relationship assumed in this study ($[\sigma_D/D]_{int} = 0.4$) is also presented using dashed lines. With this conservative choice we take into account that sizes and velocity are measured in the high redshift universe with greater uncertainties. Within the interval centered at $V = 200 \pm 20$ km s^{-1} (mean physical galaxy dimension $\sim 10 \pm 1.5$ h^{-1} kpc which roughly corresponds to an isophotal diameter $D_{25} \sim 20 \pm 3$ h^{-1} kpc), we will select galaxies with total absolute I magnitude $M_I - 5\log h = -22.4$ (see Paper II). The velocity selected standard rods at $z \sim 1$ are thus well within the visibility window of deep galaxy redshift surveys such as the VVDS (Le Fèvre et al. 2005) or zCOSMOS (Lilly et al. 2006). For exemple the VVDS is flux limited at $I < 24$ and selects objects brighter then $M_I \sim -20 + 5\log h$ at $z = 1$.

multi-slit devices, rather than get sparser samples using a single-object, near-IR spectrograph.

A detailed study of the kinematical information encoded in the [OII] line are presented in Paper II. In that paper we have explored the degree of correlation of optical H$_\alpha$ and [OII] rotational velocity indicators, i.e. how well the rotation velocities extracted from these different lines compare. Moreover, we have derived a local diameter-velocity relationship, and we have investigated the amplitude of the scatter in the zero-point calibration of the standard rods.

We finally note that the present-day expansion rate sets the overall size and time scales for most other observables in cosmology. Thus, if we hope to seriously constrain other cosmological parameters it is of vital importance either to pin down its value or to devise H_0-independent cosmological tests. Note that, given the calibration of the diameter–linewidth relation in the form $H_0 = f(V)$, the θ-expression in Eq. (3) is effectively independent of the value of the Hubble constant.

4. Optimal test strategies

In this section we outline the optimal observational strategy required in order to perform the proposed test. With the proposed selection technique, the photometric standard rod D is spectroscopically selected and the sample is therefore free from luminosity-size selection effects, that is from the well known tendency to select brighter and bigger objects at higher redshifts (Malmquist bias) in flux-limited samples. However it is crucial that a large sample of spectra be collected, in order to obtain $\sqrt{N}$ gain over the intrinsic scatter in the calibrated V(OII)-diameter relationship.

4.1. Galaxy sizes measurement

Since galaxies do not have sharp edges, their angular diameter is usually defined in terms of isophotal magnitudes. However since surface brightness is not constant with distance, the success in performing the experiment revolves around the use of metric rather than isophotal galaxy diameters (Sandage 1995).

A suitable way to measure the photometric parameter θ, without making any a-priori assumption about cosmological models, consists in adopting as the standard scale length estimator either the half-light radius of the galaxy or the η-function of Petrosian (1976). The Petrosian radius is implicitly defined as

$$\eta(\theta) = \frac{\langle\mu(\theta)\rangle}{\mu(\theta)}, \tag{4}$$

i.e. as the radius θ at which the surface brightness averaged inside θ is a predefined factor η larger than the local surface brightness at θ itself.

Both these size indicators are independent of K-correction, dust absorption, luminosity evolution (provided the evolutionary change of surface brightness is independent of radius), waveband used (if there is no color gradient) and source light profile (Djorgovski & Spinrad 1981).

4.2. Standard rods optimal selection

The choice of the objects for which the velocity parameter V and the metric size is to be measured is a compromise between the observational need of detecting high signal-to-noise spectral and photometric features (i.e. selecting high luminosity and large objects) and the requirement of sampling the velocity distribution function ($n(V)dV \sim V^{-4}$ for galaxy-scale halos) within an interval where the rotator density is substantial.

Given the estimated source of errors (see next section), and the requirement of determining both Ω_Q and w with a precision of 20%, we find, guided by semi analytical models predicting the redshift distribution of rotators (i.e. Narayan & White 1988; Newman & Davis 2000), that an optimal choice are $V = 200$ km s^{-1} rotators.

In particular, as shown in Paper II, the I band characteristic absolute magnitude of the $V = 200$ km s^{-1} objects is $M_I \sim -22.4 + 5\log h_{70}$ i.e. well above the visibility threshold of flux-limited surveys such as zCOSMOS or VVDS (as an example, for $I_{ab} = 24$ the limiting magnitude of the VVDS, one obtains that $M_I \lesssim -21 + 5\log h_{70}$ at $z = 1.5$). Therefore, with this velocity choice, the selected standards do not suffer from any Malmquist bias (i.e. any effect which favors the systematic selection of the brighter tails of a luminosity distribution at progressively higher redshifts).

4.3. The zCOSMOS potential

In Sect. 2 we have emphasized that an optimal strategy to study the expansion history of the universe consists in probing, with the angular diameter test, the $0 \le z \le 2$ interval. However, in order to collect a large and minimally biased sample of standard rods over such a wide redshift baseline, we need the joint availability of high quality images and high resolution multi-object spectra.

Space images allow a better determination of galaxy structural parameters (sizes, luminosity, surface brightness, inclination etc.) at high redshift. In particular, the ACS camera of the Hubble Space Telescope can survey large sky regions with the key advantages of a space-based experiment: diffraction limited images no seeing blurring, and very deep photometry. Ground multi-object spectrographs operating in high resolution mode allow a better characterisation of the gravitational potential-well of galaxies, facilitating the fast acquisition of large samples of standard rods. For example, in the spectroscopic resolution mode $R = 2500$ (1″ slits), the VIMOS spectrograph (Le Fèvre et al. 2003) allows to resolve the internal kinematics of galaxies via their rotation curve or line-width. Note that the VIMOS slitlets can be tilted and aligned along the major axis of the galaxies in order to remove a source of potentially significant error in the estimate of the rotation velocities.

These observational requirements are mandatory for a succesfull implementation of the proposed cosmological probe. The practical feasibility of the strategy is graphically illustrated in Fig. 3. In this figure we show a high resolution spectra of a galaxy at $z = 0.5016$ obtained with a total exposure of 90 min with VIMOS. The ground and space (ACS) images of the galaxy are also shown for comparison.

Interestingly, a large sample of rotators can be quickly assembled by the currently underway zCOSMOS deep redshift survey, which uses the VIMOS multi-object spectrograph at the VLT to target galaxies with ACS photometry in the 2 sq. degree COSMOS field Scoville et al. 2003). In principle, one can measure in high resolution modality $R = 2500$ the line-widths of [OII]$\lambda 3727$ Å up to $z \sim 1.4$ by re-targeting objects for which the redshift has already been measured in the low-resolution ($R = 600$) zCOSMOS survey. Since the spectral interval covered in high resolution mode is limited to ~2000 Å, this allows us to determine the optimal telescope and slitlets position angles in order to maximize the number of spectra whose OII emission lines fall onto the CCDs. With this fast follow-up strategy one could be able to target about 200 rotators per pointing with an exposure time of 4 h down to $I_{ab} = 24$.

In the following we will use the zCOSMOS sample as a test case to assess the performances of the proposed cosmological test.

5. Constraints on cosmological parameters

In this section we present in detail some merits and advantages of the proposed approach for constraining the value of the fundamental set of cosmological parameters. We evaluate the potential of the test, within the zCOSMOS operational specifications, in placing constraints not only on the simplest models of the universe, which include only matter and a cosmological constant, but also on so-called "quintessence" models (Turner & White 1997; Newman et al. 2002), For the purposes of this study, we assume in this section w to be constant in time up to the redshift investigated $z \sim 1.4$.

We assume that a V(OII)-diameter relation can be locally calibrated, and that the diameter of $V = 200$ km s^{-1} rotators may be inferred with a worst(/optimal) case relative error $[\sigma_D/D]_{int} = 40\%(/15\%)$ (see Sect. 3 and Fig. 2). The main contributions to this error figure are uncertainties in measuring linewidths, galaxy inclinations and the intrinsic scatter of the empirical relation itself. We then linearly combine this intrinsic scatter with the uncertainties with which the Petrosian or half-light radius may be determined from the ACS photometry. The error on the half light radii is about $\sigma_\theta = \pm 0.04″$ almost independent of galaxy sizes in the range 0.1–1″ (S. Gwyn, private communication).

We consider as the observable of the experiment the logarithm of the angle subtended by a standard rod. We use the logarithms of the angles rather than angles themselves because we assume that the object magnitudes, rather than diameters, are normally distributed around some mean value. Moreover, in this way the galactic diameter becomes an additive parameter, whose fitted value (when a $z = 0$ calibration is not available) does not distort the cosmological shape of the $\theta(z)$ function.

The observed values $\log \theta$ are randomly simulated around the theoretical value $\log \theta^r$ (cf. Eq. (3)) using the standard deviation given by

$$\sigma = \left[\left(\frac{\sigma_D}{D} \right)_{int}^2 + \left(\frac{\sigma_\theta}{\theta} \right)_{obs}^2 \right]^{1/2}. \tag{5}$$

For the purposes of this section, θ^r is computed assuming a flat, vacumm dominated cosmology with parameters $\Omega_m = 0.3$, $\Omega_Q = 0.7$, and $w = -1$ as reference model. The confidence with which these parameters are constrained by noisy data is evaluated using the χ^2 statistic

$$\chi^2 = \sum_i \frac{[\log \theta_i - \log \theta_i^{th}(z, \boldsymbol{p})]^2}{\sigma^2}, \tag{6}$$

where θ^{th} is given by Eq. (3).

We also derive the expected redshift distribution of galaxies having circular velocity in the $180 \le V \le 220$ km s^{-1} range, in various cosmological scenarios within the framework of the Press & Schechter formalism (Narayan & White 1988; Newman & Davis 2000). We note that due to the correlation between circular velocity and luminosity, these galaxies could be observed to the maximum depth ($z \sim 1.4$) out to which the OII is within the visibility window of VIMOS. We take into account the uncertainties in the semi-analytic predictions and our ignorance about the fraction of discs to be observed that will have a spectroscopically resolved [OII]$\lambda 3727$ Å line, by multiplying the calculated halo density by a "conservative" factor $f = 0.2$. Using the VIMOS $R = 2500$ resolution mode data, we thus expect to be able to implement our angular-diameter research program using nearly 1300 standard rods per square degree, which is what has been simulated (see Fig. 4).

Since we do not make a-priori assumptions about any parameter, and in particular we do not assume a flat cosmology, results should be distributed as a χ^2_ν with $\nu = 3$ degrees of freedom, which can be directly translated into statistical confidence contours, as presented in Fig. 5. This figure shows that even without assuming a flat cosmology as a prior and considering a diameter-linewidth relationship with a 40% scatter, by targeting ~1300 rotators we can directly infer the presence of a dark energy component with a confidence level better than 3σ. At the same time, its equation of state can be constrained to better than 20% (~10% if the the diameter-linewidth relationship is calibrated with a ~15% relative precision).
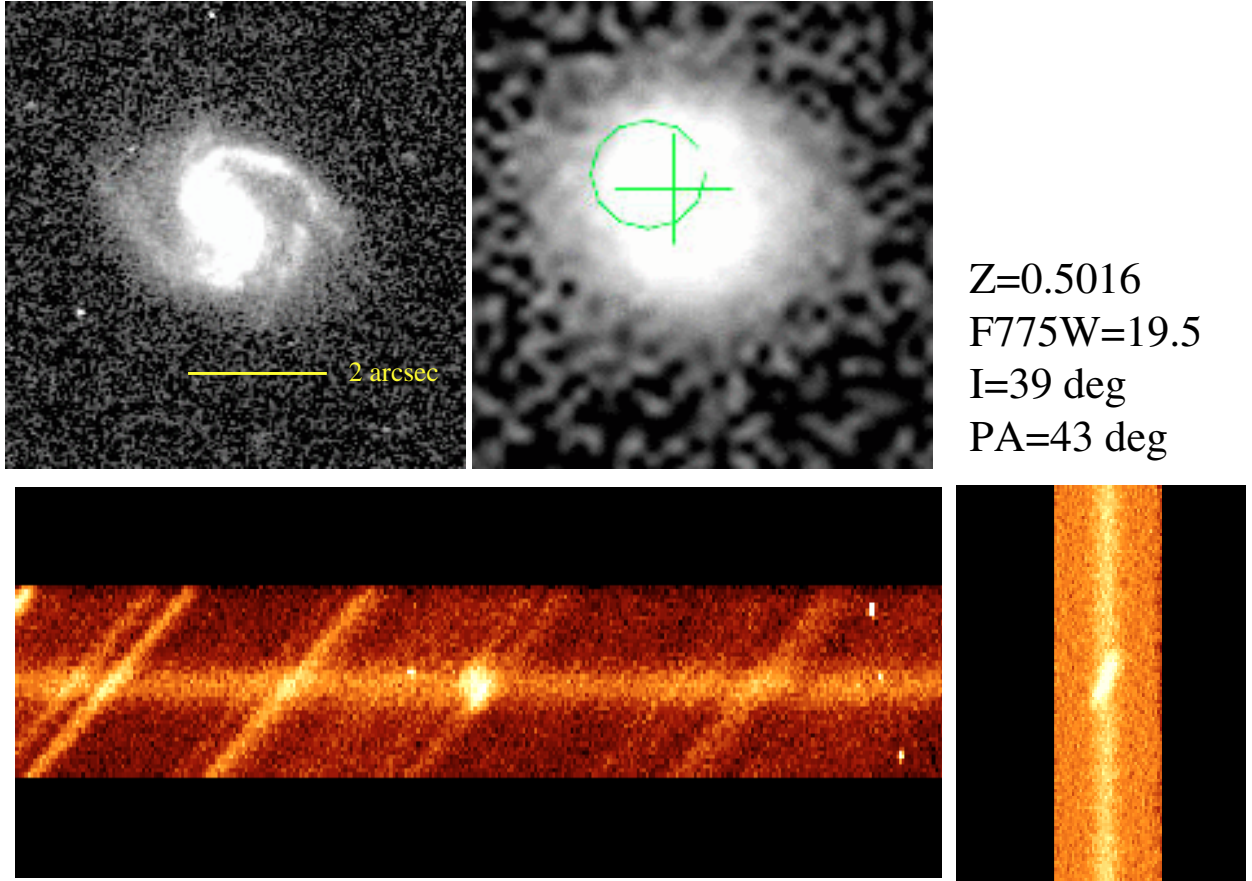
Z=0.5016
F775W=19.5
I=39 deg
PA=43 deg

Fig. 3. *Upper*: public release image of the galaxy $\alpha = 53.1874858$, $\delta = -27.910975$ at redshift 0.5016, taken with the filter *F775W* (nearly *I* band) of the ACS camera by the GOODs program. For comparison the same galaxy as imaged in the EIS survey with the WFI camera at the ESO 2.2mt telescope at La Silla. *Botton left:* raw spectrum of the galaxy taken by VIMOS at the VLT-UT3 telescope with a total exposure time of 90 min and a spectral resolution $R = 2500$. Slitlets have been tilted according to the major axis orientation (Position Angle = 43°). *Botton right:* final processed spectrum showing the rotation curve as traced by the Hb ($\lambda 4859$ Å) line.

6. Standard rod evolution

The previous analysis shows that the angular diameter test, when performed using fast high resolution follow-up of zCOSMOS spectroscopic targets may be used as a promising additional tool to explore the cosmological parameter space and directly measure a dark energy component. However, the impact of any standard rod evolution on these results needs to be carefully examined.

First of all, we may note that the expected variation with cosmic time of the total galaxy luminosity due to evolution in its stellar component does not affect the metric definition of angular diameters unless this luminosity change depends on radius (see Paper III of this series, marinoni et al. 2007, for a detailed analysis of this issue). Moreover, we can check each galaxy spectrum or image for peculiarities indicating possible evolution or instability of the standard rod which may be induced by environmental effects, interactions or excess of star formation.

Any possible size evolution of the standard rod needs to be taken into account next. Interestingly, it has been shown by different authors that large discs in high redshift samples evolve much less in size than in luminosity in the redshift range $0 < z < 1$. Recent studies show that the amount of evolution to $z \sim 1$ appears to be somewhat smaller than expected: disc sizes at $z \sim 1$ are typically only slightly smaller than sizes measured locally (Takamiya 1999; Faber et al. 2001; Nelson et al. 2002; Totani et al. 2002;

Somerville et al. 2006). This is also theoretically predicted by simulations; Boissier & Prantzos (2001), for example, show that large discs (i.e. fast rotators) should have basically completed their evolution already by $z \sim 1$ and undergo very little increase in size afterwards. Infall models (e.g., Chiappini et al. 1997; Ferguson & Clarke 2001; Bouwens & Silk 2002) also predict a mild disk size evolution. Disk sizes at $z \sim 1$ in these models are typically only 20% smaller than at $z = 0$.

6.1. Analysis of the biases introduced by evolution

Even if literature evidences are encouraging, we have to be aware that even a small amount of evolution may introduce artificial features and bias the reliability of the cosmological inferences.

In this section we directly address this issue by considering different evolutionary patterns for the standard rods, and by analyzing to what level the simulated true cosmological model may still be correctly inferred using evolved data. In other words, we investigate how different disc evolutionary histories affect the determination of cosmological parameters by answering to the following three questions:

a) is there a feature that may be used to discriminate the presence of evolution in the data?
b) which cosmological parameter is more sensitive to the eventual presence of disc evolution?, and in particular what are the
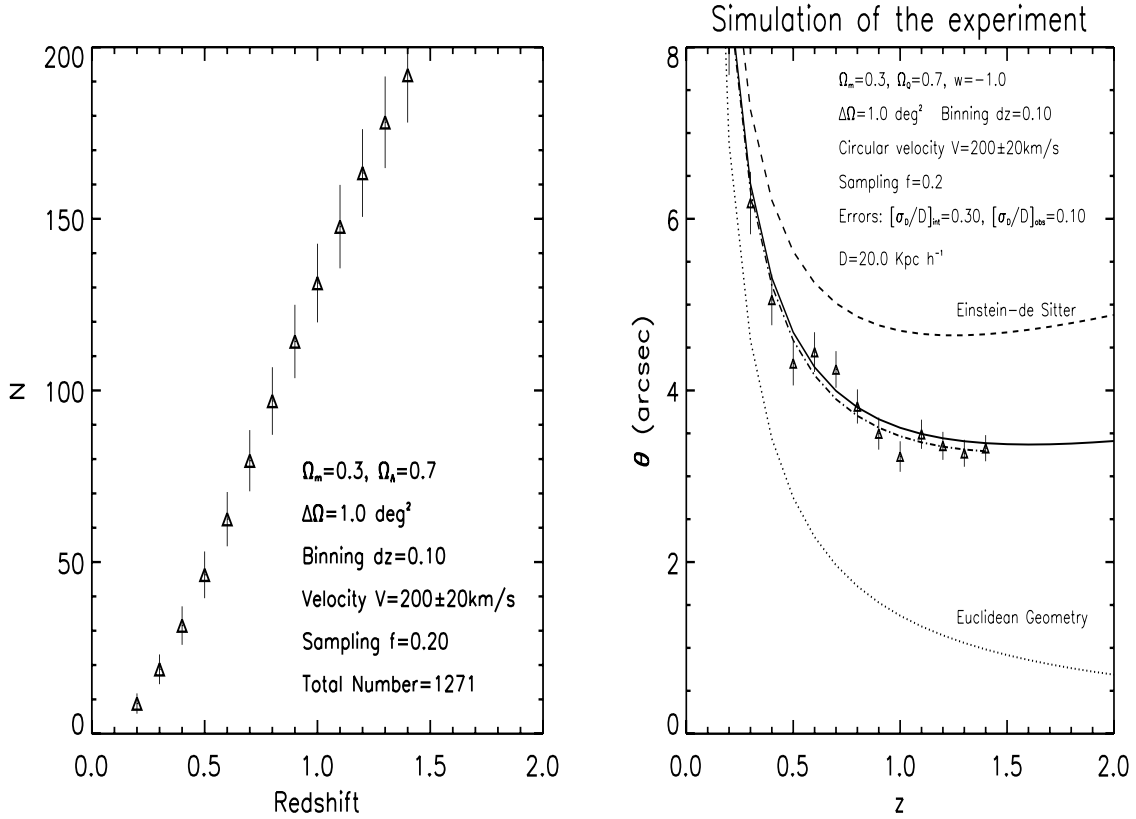
Fig. 4. *Left*: redshift evolution of the differential comoving number density of halos with a circular velocity of 200 km s^{-1} computed according to the prescriptions of Newman & Davis (2000) in the case of a flat cosmological model having $\Omega_m = 0.3$ today and a cosmological constant. Only a fraction $f = 0.2$ of the total predicted abundance of halos (i.e. ~1300 objects per square degree) is conservatively supposed to give line-width information useful for the angular-diameter test. *Right*: simulation of the predicted scatter expected to affect the angular diameter-redshift diagram should in principle achieve with the angular diameter test. The simulation is performed assuming the sample is composed by ~1300 rotators with $V = 200$ km s^{-1} and that a flat model with parameters $[\Omega_m = 0.3, \Omega_Q = 0.7, w = -1]$ is the true underlying cosmological framework. The circular velocity has been converted into an estimate of the galaxy diameter ($D_v = 20$ kpc) by using the velocity-diameter template calibrated by Bottinelli et al. (1980). The *worst-case* scenario ($[\sigma_D/D]_{int} = 40\%$) is presented. The solid line visualizes the underlying input cosmological model $\theta^{\Lambda CDM}(z)$, while triangles are drawn from the expected Poissonian fluctuations. The dot-dashed line represents the expected scaling of the angular diameter in our best recovered cosmological solution. The dashed and the dotted lines represent the angular scaling in a Einstein-de Sitter and Euclidean (non-expanding type cosmology with zero curvature) geometry respectively.

effects of evolution on the value of the inferred dark energy density parameter Ω_Q?

c) is there a particular evolutionary scenario for which the inferred values of Ω_Q and w are minimally biased?

For the purposes of this analysis, we consider for the angular diameter-redshift test the baseline ($0.1 \leq z \leq 1.4$) divided in bins of width d$z = 0.1$ and assume a relative scatter in the mean size per bin of 5%. This scatter nearly corresponds to that expected for a sample of 1300 rotators (with $0 < z < 1.4$ and d$z = 0.1$) whose diameters are individually (and locally) calibrated with a 40% precision. We then select a given fiducial cosmology (input cosmology), apply an arbitrary evolution to the standard rods, and then fit the evolved data with the unevolved theoretical prediction given in Eq. (3) in order to obtain the best fitting (biased) output cosmology and the associated confidence levels contours. We decide that the best fitting cosmological model offers a good fit to the evolved data if the probability of a worse χ^2 is smaller that 5% (i.e. $P(\chi^2 > \chi^2_{obs}) < 0.05$).

We adopt three different parameterizations to describe an eventual redshift evolution of the velocity selected sample of galaxy discs D_v: a *late-epoch* evolutionary scenario ($\Delta D/D \equiv (D_v(z) - D_v(0))/D_v(0) = -|\delta_1| \sqrt{z}$) where most of the evolution

is expected to happen at low redshifts and levels off at greater distances (δ_1 is the relative disc evolution at $z = 1$), a *linear* evolutionary scenario ($\Delta D/D = -|\delta_1|z$) without any preferred scale where major evolutionary phenomena take place (i.e. the gradient of the evolution is nearly constant), and an *early-epoch* evolution scenario ($\Delta D/D = -|\delta_1|z^2$) where most of the evolution is expected to happen at high redshift.

We note that for modest disc evolution, the linear parameterization satisfactorily describes the whole class of evolutionary models whose series expansion may be linearly represented (for example, the hyperbolic model ($D_v(z) = D_v(0)/(1 + |\delta_1|z)$)). For $z \ll 1$ it also represents fairly well the exponential model ($D_v(z) = D_v(0)(1-z)^\delta$). Moreover, the linear model is the favored scenario for disc size evolution at least at low redshift ($\lesssim 1.5$) as predicted by simulations (e.g. Mo et al. 1998; Bouwens & Silk 2002).

First, let's assume that $w = -1$ and that the dark energy behaves like Einstein's cosmological constant. In Fig. 6 we consider three different input fiducial cosmological models (a flat Λ-dominated universe ($\Omega_\Lambda = 0.7$), an open model ($\Omega_m = 0.3$) and an Einstein-de Sitter universe ($\Omega_m = 1$)) and show the characteristic pattern traced by the best fitting output values
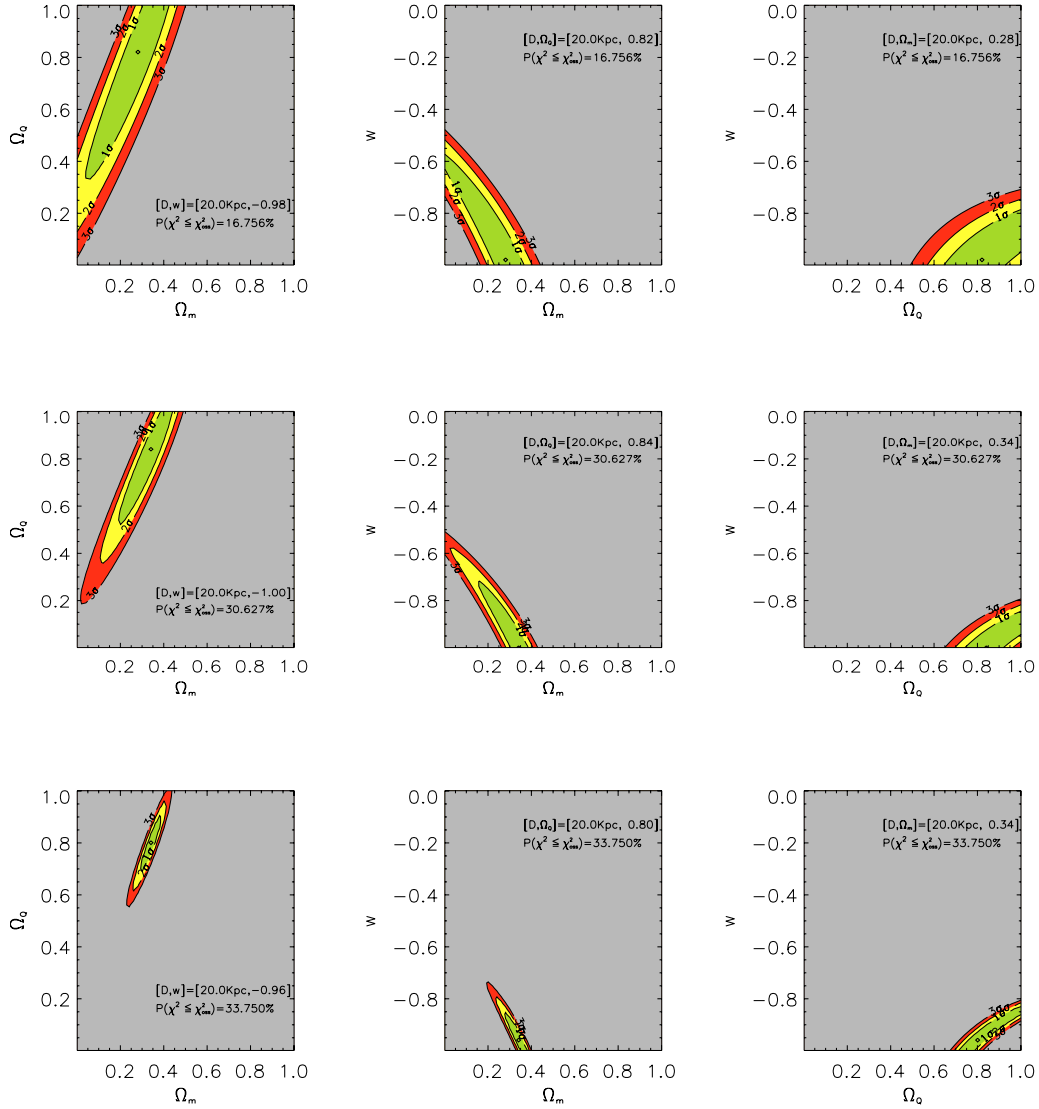
Cosmological Constraints

Fig. 5. Predicted 1, 2, and 3σ confidence level contours for application of the angular-diameter test. The likelihood contours have been derived by adopting a Λ-cosmology $[\Omega_m, \Omega_Q, w] = [0.3, 0.7, -1]$ as the fiducial model we want to recover, and by conservatively assuming one can obtain useful line-widths information for a sample of galaxies having the redshift distribution and the Poissonian diameter fluctuations simulated in Fig. 4. Confidence contours are projected onto various 2D-planes of the $[\Omega_m, \Omega_Q, w]$ parameter space, and the jointly best fitted value along the projection axis, together with the statistical significance of the fit, are reported in the insets. Note the strong complementarity of the confidence region orientation which is orthogonal to the degeneration axis of the CMB measurements. *Top*: constraints derived assuming that one might survey only 1 square degree of sky and that the V(OII)-diameter relation is locally calibrated with a 40% of relative scatter in diameter. *Center*: as before but assuming a scatter of 15% in diameters. *Bottom*: confidence contours for a survey of 16 deg^2 (which corresponds to the full area surveyed by VIMOS-VLT Deep Survey) assuming a template V(OII)-diameter relationship with a scatter of 30% in diameters.

(Ω_m, Ω_Λ) inferred by applying the angular diameter test to data affected by evolution. A common feature of all the various evolutionary schemes considered is that the value of Ω_m is systematically underestimated with respect to its true input value: the stronger the evolution in diameter and the smaller Ω_m will be, irrespectively of the particular disc evolutionary model considered. Since many independent observations consistently indicate the existence of a lower bound for the value of the normalized matter density ($\Omega_m \gtrsim 0.2$), we can thus use this parameter as a sensitive indicator of evolution.

Once the presence of evolution is recognized, the remaining problem is to determine the level of bias introduced in the dark

energy determination. If the gradient of the disk evolution function increases with redshift (*quadratic* evolution), then the estimates of Ω_Λ are systematically biased low. The contrary happens if the evolutionary gradient decreases as a function of look-back time (*square root* model). If the disc evolution rate is constant (*linear* model), then even if discs are smaller by a factor as large as 40% at $z = 1.5$ the estimate of the dark energy parameter is only minimally biased. The net effect of a linear evolution is to approximately shift the best fitting Ω_Λ value in a direction parallel to the Ω_m axis in the $[\Omega_m, \Omega_\Lambda]$ plane.

More generally, by linearly evolving disc sizes so that they are up to 40% smaller at $z = 1.5$, and by simulating the apparent
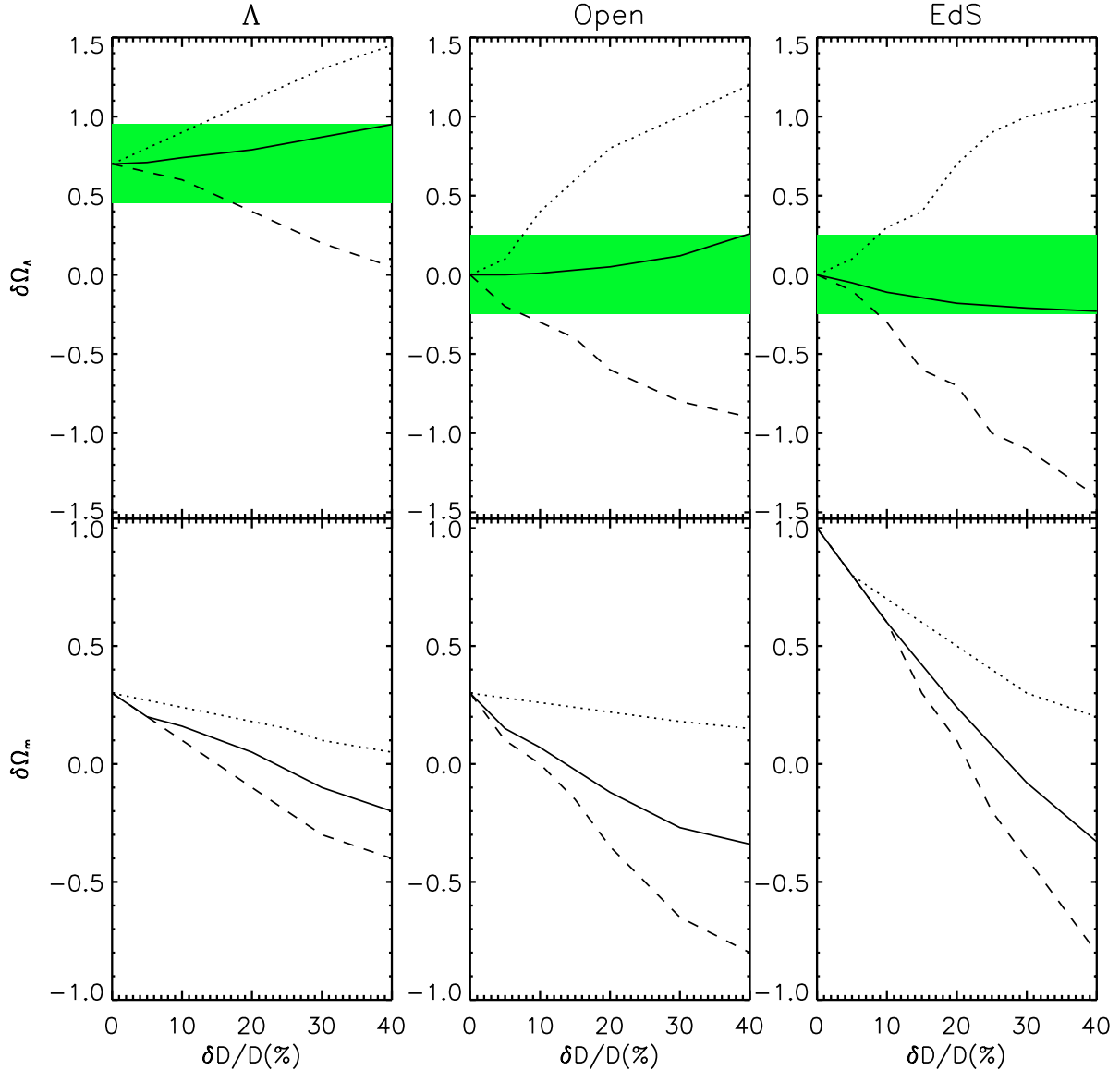
Fig. 6. Best fitting cosmological parameters inferred by applying the angular diameter test to data affected by evolution. The output (biased) estimates of Ω_m and Ω_Λ are plotted as a function of the relative diameter evolution for the following evolutionary models: linear (solid line), square-root (dotted line) and quadratic evolution (dashed line). The biasing pattern is evaluated for three different fiducial cosmologies: a flat, Λ-dominated cosmology ($\Omega_\Lambda = 0.7$, *left*), a low-density open cosmology ($\Omega_m = 0.3$, *center*) and a flat, matter-dominated model ($\Omega_m = 1$, *right*).

angle observed in any arbitrary cosmological model with matter and energy density parameters in the range $0 < \Omega_m < 1$ and $0 \leq \Omega_\Lambda \leq 1$ ($w = -1$) we conclude that the maximum deviation of the inferred biased value of Ω_Λ from its true input value, is limited to be $|\mathrm{max}(\delta\Omega_\Lambda)| \lesssim 0.2$, whatever the true input value of the energy density parameter is. In other terms, in the particular case of a linear and substantial ($<40\%$ at $z = 1.5$) evolution of galaxy discs, the central value of the dark energy parameter is minimally biased for any fiducial input model with $0 \leq \Omega_m < 1$ and $0 \leq \Omega_\Lambda \leq 1$).

We have shown that the presence of evolution is unambiguously indicated by the "unphysical" best fitting value of the parameter Ω_m. We now investigate the amplitude of the biases induced by disc evolution in the $[\Omega_Q, w]$ plane. We assume for this purpose that w is free to vary in the range $-1 \leq w \leq -1/3$, which means assuming that the late epoch acceleration of the universe might be explained in terms of a slow rolling scalar field.

We first consider a situation where the disc size evolution is modest, and could be represented by any of the three models considered. Whatever the mild evolution model considered (less then 15% evolution from $z = 1.5$) and assuming a scatter in the angular diameter-redshift diagram of 5% in each redshift bin, we find that the input values of Ω_Q and w are contained within the 1σ biased confidence contour derived from the evolved data.

Figure 7 shows the 1, 2 and 3σ "biased" confidence contours obtained by fitting with Eq. (3) a simulated angular-diameter redshift diagram in which discs have been linearly evolved. We show that, *if disc evolution depends linearly on redshift and causes galaxy dimensions to be up to 30% smaller at $z = 1.5$, the true fiducial input values of Ω_Q and w are still within 1σ of the biased confidence contours inferred in presence of a standard rod evolution* (and a scatter in the angular diameter-redshift relation as low as 5% in each redshift bin). We have tested that these conclusions hold true for every fiducial input cosmology
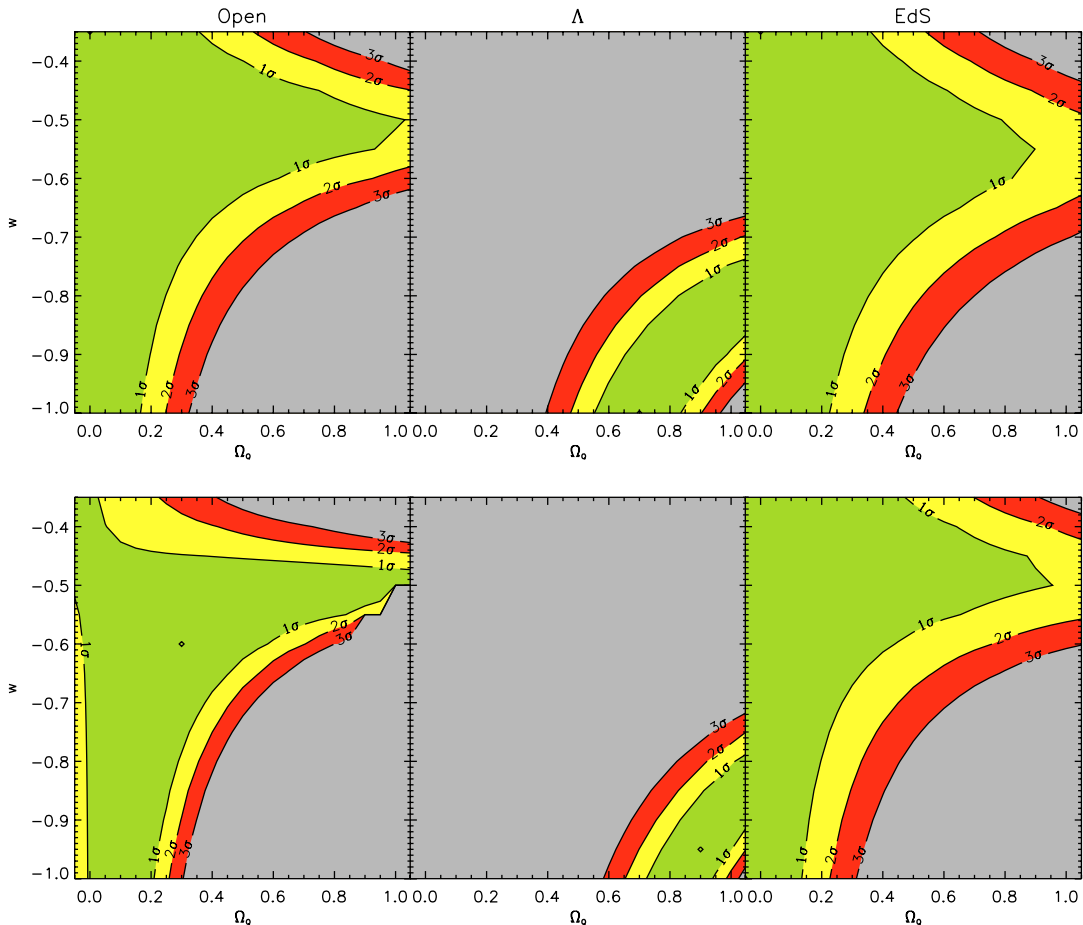
Fig. 7. 1, 2, and 3σ confidence level contours in the $[\Omega_Q, w]$ plane computed by applying the angular-diameter test to data unaffected (*upper panel*) and affected by diameter evolution (*lower panel*). We consider a linear model for disc evolution normalized by assuming that discs were smaller by 30% at $z = 1.5$, and a nominal relative error in the standard rod measures of 5% per redshift bin (see discussion in Sect. 6.1). The effects of disc evolution onto cosmological parameter estimation are compared to the evolution-free case for three different fiducial cosmologies: a low matter density open cosmology ($\Omega_m = 0.3$, *left*) a flat, Λ-dominated cosmology ($\Omega_\Lambda = 0.7$, *center*), and an Einstein-de Sitter model ($\Omega_m = 1$, *right*).

with parameters in the range $0 \leq \Omega_m \leq 1$, $0 \leq \Omega_Q \leq 1$ and $-1 \leq w \leq -1/3$.

Thus, if, disc evolution is linear (as predicted by theoretical models) and substantial (up to ~30% at $z = 1.5$), or arbitrary and mild (up to ~15% at $z = 1.5$), then in both cases the angular diameter test reduces from a test of the whole set of cosmological parameters, to a direct and fully geometrical test of the parameters subset (Ω_Q, w). For example, in a minimal approach, the angular diameter test could be used to test in a purely geometrical way the null hypothesis that "*a dark energy component with a constant equation of state parameter w is dominating the present day dynamics of the universe*". Moreover, as Fig. 7 shows, an universe dominated by dark energy may be satisfactorily discriminated from a matter dominated universe ($\Omega_Q = 0$).

In the evolutionary pictures considered, galaxy discs are supposed to decrease monotonically in size in the past. Since the sensitivity of the test to changes in the linear diameter D is described by a growing monotonic function in the redshift interval [0, 1.4], one may hope to test cosmology in a way which is less dependent on systematic biases by limiting the sample at $z \leq 1$. However, by doing this we would halve the number of standard rods available for the analysis (~600 with respect to the original ~1300). The test efficiency in constraining cosmological parameters would consequently be significantly degraded.

6.2. The Hubble-diagram using galaxies

The same velocity criterion that allows the selection of standard rods also allows the selection of a sample of standard candles. Using the same set of tracers, the Tully & Fisher relationships connecting galactic rotation velocities to luminosities and sizes offer the interesting possibility of implementing two different cosmological tests, the angular-diameter test and the Hubble diagram.

Thus, we have analyzed how the effects of luminosity evolution may bias the estimation of cosmological parameters in the case of the Hubble diagram. Assuming that the absolute luminosity L of the standard candle increases as a function of redshift according to the square root, linear and quadratic scenarios, then the value of Ω_m is systematically overestimated. The value of Ω_m is biased in an opposite sense with respect to the angular diameter test (see Fig. 6). Therefore, it is less straightforward to discriminate the eventual presence of evolution in the standard candle on the basis of the simple requirement that any "physical" matter density parameter is characterized by a positive lower bound. The different Ω_m shifts (with respect to the fiducial value) observed when the evolved data are fitted using the Hubble diagram or the angualr diameter test are due to the fact that, given the observed magnitudes and apparent angles, an increase with redshift in the standard candle absolute

luminosity causes the best fitting distances to be biased towards higher values, while a decrement in the physical size implies that real cosmological distances are underestimated.

Moreover, even considering a linear evolutionary picture for the absolute luminosity as well as a modest change in the standard candle luminosity, i.e. $\Delta_M = M(z) - M(0) = -0.5$ at $z = 1.5$, the input fiducial cosmology falls outside the 3σ confidence contour obtained by applying the magnitude-redshift test to the sample of evolved standard candles. Note that error contours are derived by assuming a scatter of $\sigma_M = 0.05$ per redshift bin in the Hubble diagram. Since, at variance with the size of large discs, galactic luminosity is expected to evolve substantially with redshift (within the VVDS survey, $\Delta_M \sim -1$ in the I band for M* galaxies Ilbert et al. 2005) we conclude that the direct implementation of the Hubble diagram test as a minimal test for the parameter subset $[\Omega_Q, w]$ using galaxy rotation as the standard candle indicator is more problematic. As an additional problem, we note that galaxy luminosity is seriously affected by uncertainties in internal absorption corrections, and that due to K-correction, the implementation of the Hubble diagram requires multi band images to properly describe the rest frame emission properties of galaxies.

6.3. The cosmology-evolution diagram

In this section we want to address the more general question of whether it is possible to infer cosmological information knowing a-priori only the upper limit value for disc evolution at some reference redshift (for example, the maximum redshift surveyed by a given sample of rotators). In other terms, we explore the possibility of probing geometrically the cosmological parameter space in a way which is independent of the specific evolution function with which disc sizes change as a function of time. The only external prior is the knowledge of an upper limit for the amplitude of disc evolution at some past epoch.

Given an arbitrary model specified by a set of cosmological parameters $\boldsymbol{p}$, and given the observable $\theta^{\text{obs}}(z)$, i.e. the apparent angle subtended by a sample of velocity selected galaxies (with locally calibrated diameters $D_v(0)$), then

$$\epsilon_\theta(z, \boldsymbol{p}) = \theta^{\text{obs}}(z) - \theta^{\text{th}}(D_v(0), z, \boldsymbol{p}) \tag{7}$$

is the function which describes the redshift evolution of the standard rod, i.e. $\epsilon_\theta = (D_v(z) - D_v(0))/d_\theta$ in the selected cosmology.

Let's suppose that we know the lower and upper limits of the relative (adimensional) standard rod evolution ($\delta(\bar{z}) = \frac{|\Delta D_v(\bar{z})|}{D_v(0)}$) at some specific redshift $\bar{z}$.

Assuming this prior, we can solve for the set of cosmological parameters (i.e. points $\boldsymbol{p}$ of the cosmological parameter space) which satisfy the condition

$$\delta_l(\bar{z}) \le \frac{|\epsilon_\theta(\boldsymbol{p}, \bar{z})|}{\theta^{\text{th}}(D(0), \bar{z}, \boldsymbol{p})} \le \delta_u(\bar{z}). \tag{8}$$

This inequality establishes a mapping between cosmology and the amount of disc evolution at a given redshift which is compatible with the observed data. By solving it, one can construct a self-consistent cosmology-evolution plane where to any given range of disc size evolution at $\bar{z}$ corresponds in a unique way a specific region of the cosmological parameter space. Vice versa, for any given cosmology one can extract information about disc evolution. Clearly, the scatter in the angular-diameter diagram directly translates in the uncertainties associated to the evolution boundaries in the cosmological space.
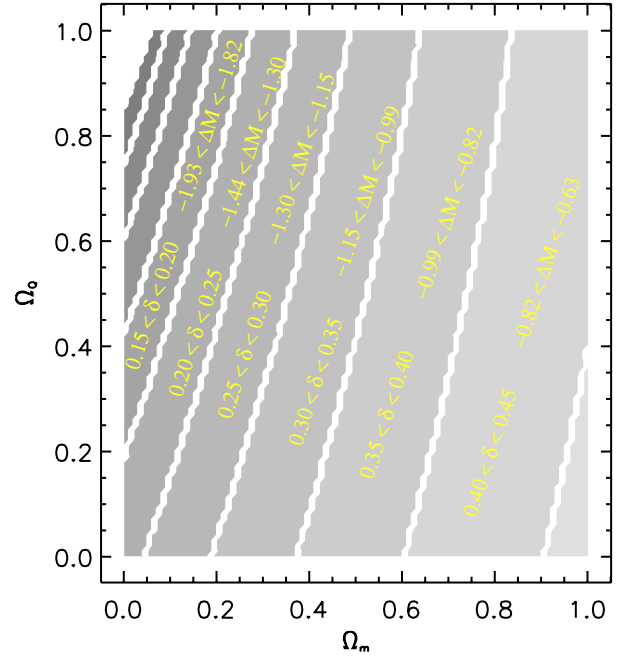

Fig. 8. Cosmology-evolution diagram for simulated data which are affected by evolution. Apparent angles and luminosities of the velocity selected sample of rotators are simulated in a ΛCDM cosmology. Standard rods and candles have been artificially evolved so that at $\bar{z} = 1.5$ discs are 26% smaller and luminosities 1.4 mag brigther. The cosmological plane is partitioned with different boundaries obtained by solving Eq. (8) for different values of $\delta(\bar{z} = 1.5)$, i.e. of the external prior representing the guessed upper limit of the relative disc evolution at the maximum redshift covered by data. The external prior is also expressed in term of absolute luminosity evolution (see discussion in Appendix A).

We note that the boundaries of the region of the cosmological parameter space which is compatible with the assumed prior on the evolution of diameters at $\bar{z}$ can be equivalently expressed in term of the maximum absolute evolution in luminosity (see Appendix A). This because, as stated in the previous section, velocity selected objects have the unique property of being at the same time standards of reference both in size and luminosity.

With this approach, one may by-pass the lack of knowledge about of the particular evolutionary track of disc scalelengths and luminosities and try to extract information about cosmology/evolution by giving as a prior only the fractional evolution in diameters or the absolute evolution in magnitude expected at a given redshift. The essence of the method is as follows: instead of directly putting constraints in the cosmological parameters space by mean of cosmological probes, we study how bounded regions in the evolutionary plane ($\frac{\Delta D}{D}, \Delta M$) map onto the cosmological parameter space.

In Fig. 8 we show the cosmology-evolution diagram derived by solving Eq. (8) for different ranges of $\delta(\bar{z})$. The reference model is the concordanace model ($\Omega_m = 0.3$, $\Omega_Q = 0.7$, $w = -1$) and the reference evolution at $\bar{z} = 1.5$ is assumed to be $\delta(\bar{z}) = 0.25$ for discs, and $M_v(\bar{z}) - M_v(0) = -1.5$ for luminosities. The cosmology-evolution diagram represents the unique correspondance between all the possible cosmological models and the amount of evolution in size and luminosity which is compatible with the observed data at the given reference redshift. Using independent informations about the range of evolution expected in the structural parameters of galaxies, for exemple from simulations or theoretical models, one may constrain the value of
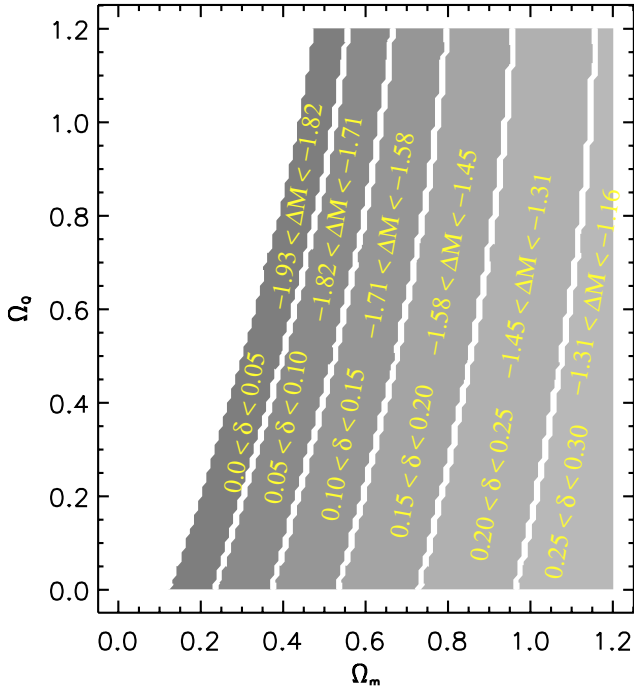
Fig. 9. As in Fig. 8 but for a different cosmology (an Einstein-de Sitter universe).

cosmological parameters. Vice versa, if the cosmological model is known, then one may directly determine the evolution in magnitude and size of the velocity selected sample of rotators.

With this approach, the possibility of discriminating between different cosmologies depends on the amount of evolution affecting the standard rods at redshift $\bar{z}$. Two different sets ($\boldsymbol{p_1}$ and $\boldsymbol{p_2}$) of cosmological parameters may be discriminated at $z = \bar{z}$ if the relative disc evolution at $\bar{z}$ is known to a precision better than

$$\delta(\bar{z}) < \frac{|r(\bar{z}, \boldsymbol{p_2}) - r(\bar{z}, \boldsymbol{p_1})|}{r(\bar{z}, \boldsymbol{p_1})}. \quad (9)$$

For example, this kind of analysis shows that an Einstein-de Sitter universe may be unambigously discriminated from a critical universe with parameter $\Omega_m = 0.3, \Omega_Q = 0.7, w = -1$. if the relative disc evolution at $\bar{z} = 1.5$ is known with a precision better than 28%. In a similar way, an open ($\Omega_m = 0.3, \Omega_Q = 0$) universe may be discriminated from an Einstein de-Sitter universe if $\delta(\bar{z}) < 17\%$.

7. Conclusions

The scaling of the apparent angular diameter of galaxies with redshift $\theta(z)$ is a powerful discriminator of cosmological models. The goal of this paper is to explore the potentiality of a new observational implementation of the classical angular-diameter test and to study its performances and limitations.

We propose to use the velocity-diameter relationship, calibrated using the [OII]λ3727 Å line-widths, as a tool to select standard rods and probe world models. As for other purely geometrical test of cosmology, a fair sampling of the galaxy population is not required. It is however imperative to have high quality measurements of the structural parameters of high redshift galaxies (disc sizes and rotational velocity). Surveys with HST imaging and high enough spectral resolution will thus provide the fundamental ingredients for the practical realization of the recipe we have presented.

In order to avoid any luminosity dependent selection effect (such as for exemple Malmquist bias) it is necessary to apply the proposed test to high velocity rotators. We show that nearly 1300 standard rods with rotational velocity in the bin $V \sim 200 \pm 20$ km s^{-1}) are expected in a field of size 1 deg^2 over the redshift range $0 < z < 1.4$. Interestingly this large sample can be quickly assembled by the currently underway zCOSMOS deep redshift survey, which uses the VIMOS multi-object spectrograph at the VLT to target galaxies photometrically selected using high-resolution ACS images.

Even allowing a scatter of 40% for the [OII]λ3727Å linewidth-diameter relationship for disc galaxies, we show that the angular-diameter diagram constructed using this sample is affected by a scatter of only ~5% per redshift bin of amplitude d$z = 0.1$. This scatter translates into a 20% precision in the "geometric" measurement of the dark energy constant equation of state parameter w, through a test performed without *priors* in the [Ω_m, Ω_Q] space.

Current theoretical models suggest that large discs (i.e. fast rotators) evolve weakly with cosmic time from $z = 1.5$ down to the present epoch. Anyway, we have explored how an eventual evolution of the velocity-selected standard rods might affect the implementation of the test. We have shown that any possible evolution in the standard rods may be unambiguously revealed by the fact that even a small decrement with redshift of the disc sizes shifts the inferred value of the matter density parameter into "a-priori excluded" regions ($\Omega_m < 0.2$).

We have shown that a linear (as expected on the basis of various theoretical models) and substantial (up to 40% over the range $0 < z < 1.5$) disc evolution minimally biases the inferred value of a dark energy component that behaves like Einstein's cosmological constant Λ. Moreover we have shown that assuming that discs evolve in a linear-like way as a function of redshift, and that their sizes were not more than 30% smaller at $z = 1.5$ with respect to their present epoch dimension, then the angular diameter test can be used to place interesting constraints in the [Ω_Q, w] plane. In particular, assuming a scatter of 5% per redshift bin in the angular diameter-redshift diagram (nearly corresponding to the scatter expected for a sample of 1300 rotators with $0 < z < 1.4$, d$z = 0.1$, whose diameter is locally calibrated with a 40% precision), we have shown that the input fiducial [Ω_Q, w] point is still within the 1σ error contours obtained by applying the angular diameter test to the evolved data.

Finally, we have outlined the strategy to derive a cosmology-evolution diagram with which it is possible to establish an interesting mapping between different cosmological models and the amount of galaxy disc/luminosity evolution expected at a given redshift. The construction of this diagram does not require an a-priori knowledge of the particular functional form of the galaxy size/luminosity evolution. By reading this diagram, one can infer cosmological information once a theoretical prior on disc or luminosity evolution at a given redshift is assumed. In particular if the amplitude of the relative disc evolution at $\bar{z} = 1.5$ is known to better than ~30%, then an Einstein-de Sitter universe ($\Omega_m = 1$) may be geometrically discriminated from a flat, vacuum dominated one ($\Omega_m = 0.3$, $\Omega_Q = 0.7$). Viceversa, one can use the cosmology-evolution diagram to place constraints on the amplitude of the galaxy disc/luminosity evolution, once a preferred cosmology is chosen.

In conclusion, given the simple ingredients entering the proposed implementation strategy, nothing, besides evolution of discs, could in principle bias the test. Even so, evolution can be easily diagnosed and, under some general conditions, it can be shown that it does not compromise the possibility of

detecting the presence of dark energy and constraining the value of its equation of state.

In the following papers of this series (Saintonge et al. 2008; Marinoni et al. 2008), we implement the proposed strategy to a preliminary sample of velocity-selected high redshift rotators.

Appendix A

Given a spectroscopically selected sample of objects with constant rotational velocity we can derive the observed magnitude $m^{\rm o}$ of a standard candle of absolute magnitude $M_v(0)$ located at redshift z, by using the standard relation Sandage (1972)

$$m^{\rm o} = m^{\rm th}(M_v(0), z, \boldsymbol{p}) + \epsilon_M(z) + K(z) \tag{10}$$

where

$$m^{\rm th} = M_v(0) + 5 \log d_{\rm L}(z, \boldsymbol{p}) + 25$$

and were d_L is the luminosity distance, $K(z)$ is the K correction term and $\epsilon_M(z)$ is the a-priori unknown evolution in luminosity of our standard candle, i.e. $\epsilon_M(z) = \Delta M_v(z) = M_v(z) - M_v(0)$ is the difference between the absolute magnitude of an object of rotational velocity V measured at redshift z and the un-evolved local standard value $M_v(0)$.

From the definition of wavelength-specific surface brightness μ we deduce that the variation as a function of redshift in the average intrinsic surface brightness (within a radius R) for our set of homologous galaxies is

$$\Delta \langle \mu^{\rm th}(z) \rangle_R = \Delta M_v(< R) - 5 \log \frac{R(z)}{R(0)}.$$

By opportunely choosing the half light radius D_v as a metric definition for the size of a galaxy we immediately obtain

$$\Delta \langle \mu^{\rm th}(z) \rangle_D = \epsilon_M(z, \boldsymbol{p}) - 5 \log \left(\frac{\epsilon_\theta(z, \boldsymbol{p})}{\theta^{\rm th}(D(0), z, \boldsymbol{p})} + 1 \right). \tag{11}$$

The intrinsic surface brightness evolution is not an observable, but in a FRW metric this quantity is related to the surface brightness change observed in a waveband $\Delta\lambda$ by the relation

$$\Delta \langle \mu^{\rm o}(z) \rangle_D = \Delta \langle \mu^{\rm th}(z) \rangle_D + 2.5 \log(1 + z)^4 + K(z). \tag{12}$$

Thus, once we measure the redshift evolution of $\Delta \langle \mu^{\rm o}(z) \rangle_D$ for the sample of rotators, the absolute evolution in luminosity corresponding to a given relative evolution in diameters can be directly inferred using Eq. (11).

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