Transport by capillary waves. Part I. Particle trajectories

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Transport by capillary waves. Part I. Particle trajectories

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The transport of particles on capillary waves generated by the Faraday instability is studied experimentally. The motion of a single particle on the fluid surface is determined over $4 \times 10^4$ wave periods by particle tracking. Erratic particle motion over large distances occurs even for small wave amplitudes, where the pattern is ordered, though spatially modulated. This implies that the wave field is accompanied by significant drift flows with velocities approximately 30 times smaller than the instantaneous local fluid velocities. The particle motion is characterized statistically. The displacements of the particle in the two orthogonal directions follow independent Gaussian distributions. It is found that the variance of the particle’s displacement grows with the elapsed time interval as $V(\tau) \sim \tau^{0.8}$ over times long compared with the correlation time of the motion. The exponent $H$ depends on the wave amplitude, decreasing gradually from about 0.7 to an asymptotic value of 0.5 (corresponding to classical Brownian motion) at large wave amplitudes.

I. INTRODUCTION

The transport of passive impurities in hydrodynamic flows is of interest for both fundamental reasons and for its relevance to practical problems. The subject has recently been rejuvenated through two theoretical developments: an appreciation of the relevance of dynamical systems to understanding transport and the application of fractal concepts to turbulent flows. In the dynamical systems approach to mixing, initiated by Aref1 and extended by various investigators,2 the development of complex trajectories is understood to arise in some cases from nonlinearity leading to the nonintegrability of the velocity field. This phenomenon can occur even when the velocity field is stationary or time periodic. The dynamical systems approach is useful mainly for flows that are spatially coherent, in the sense that autocorrelation functions decay slowly with distance. For turbulent flows, which have only short-range spatial coherence, a spot of dye becomes convoluted over a wide range of scales.3,4 Recent descriptions of turbulent mixing have emphasized the utility of fractal and multifractal concepts. For example, the surface separating the impurity from the surrounding fluid can be described as a fractal.5,6

The kinematical or dynamical systems approach has focused attention on the utility of the Lagrangian viewpoint in which the trajectories of individual fluid particles are observed or computed. These trajectories are now known to be surprisingly complex. Though there are numerous visualization experiments where particle motion has been used to understand flow phenomena,7 we are not aware of quantitative experiments on the long term trajectories of individual particles. In this paper, we present such a study for particles moving on a fluid surface. Their motion is governed by the dynamics of standing capillary waves resulting from the Faraday instability. We choose this system both because wave propagation is ubiquitous and of inherent interest, and because well-controlled laboratory studies are possible. Naturally, we hope that the basic phenomena are not excessively dependent on the method of excitation of the waves.

The Faraday instability occurs when a liquid with a free surface is subjected to vertical oscillation of the container. The surface undergoes a parametric instability and exhibits standing waves at half the driving frequency. Extensive experimental and theoretical studies have been carried out in the last decade in order to understand the nonlinear dynamical properties of these waves. The reader may refer to Miles and Henderson8 for a recent review. At sufficiently large driving frequencies, the wavelength $\lambda$ is small compared to the dimension $L$ of the system. In such a large aspect ratio system, one can neglect the complex processes of damping at the lateral boundaries of the fluid, and the resulting effect on the wave pattern. The observed pattern has square symmetry irrespective of the shape of the container.9,10 For driving amplitudes slightly above onset, the wave pattern is highly ordered and spatially coherent over distances comparable to $L$. However, for slightly larger amplitudes, a modulational instability leads to a disordered pattern. Flows of this type are termed spatiotemporally chaotic, and form an important intermediate case between low-dimensional chaotic systems and fully developed turbulence.11 The temporal and spatial disorder of the pattern can lead to complex motion of a particle on the surface, which is the subject of this paper.

It is well known12 that particle trajectories in the case of small-amplitude propagating waves in deep water are circular. More recently, particle motion in large-amplitude (nonlinear) gravity and capillary waves have been analyzed.13,14 These studies have shown that the particle trajectories need not be closed, and that significant horizontal drift velocities can occur. Hogan15 found that for nonlinear capillary waves, a surface particle may experience a horizontal drift velocity comparable to the phase velocity of the propagating...
waves, but dependent on the wave amplitude. Significant drift velocities also occur at depths comparable to the wavelength. In the case of standing waves, the particle motion is essentially vertical beneath the antinodes, and horizontal near the nodes. The surface drift velocity is extremely small for deep water standing waves. Of course, real waves may be partially propagating if the amplitudes of the oppositely directed components are unequal. Furthermore, the wave amplitudes may be slowly modulated in space or time, and may vary in two dimensions. The effects of these factors on particle transport are unknown.

In some situations complex motion of particles may occur, and statistical analysis of the particle trajectories may yield some insight into the transport process. A useful starting point is to model the quasirandom motion of the particles as Brownian. If \( B(t) \) denotes the position of the particle along an arbitrary axis, then the probability distribution \( P(\Delta B(\tau)) \) of the displacement \( \Delta B(\tau) = B(t+\tau) - B(t) \) over a time interval \( \tau \) is expected to be Gaussian. Brownian motion also has an important invariance property with respect to a transformation in which time is scaled by a factor \( b \) and displacements are scaled by a factor \( b^{-1/2} \). The displacement \( \Delta B(\tau) \) and the scaled displacement \( b^{-1/2} \Delta B(b \tau) \) have identical probability distributions. Such a transformation, where the scaling of \( \tau \) and \( \Delta B(\tau) \) are different, has been called an "affinity" and the Brownian process is said to be "self-affine." Finally, the variance \( V(\tau) \) of the particle displacements increases linearly with time and can be written as \( V(\tau) = 2D\tau \), where \( D \) is an effective diffusion constant.

However, various types of anomalous transport occur in hydrodynamic systems, where the variance of the particle displacement does not grow linearly with time, leading to a time-dependent apparent diffusivity \( D(t) \). For instance, this occurs as a consequence of chaotic advection where a particle may be trapped for long times in subdomains of the flow.\(^{17-20}\) Anomalous diffusion also occurs in porous media and in models of arrested shear flows that have many stagnant side branches.\(^{21-23}\)

Processes of this type can be modeled using a generalization of Brownian motion known as \textit{fractional} Brownian motion.\(^{24,25}\) For fractional Brownian motion, \( B_H(t) \), the displacement \( \Delta B_H(\tau) \) and the scaled displacement \( b^{-H} \Delta B_H(b \tau) \) are identical in distribution. The variance \( V(\tau) \) of the displacement \( \Delta B_H(\tau) \) follows the power law
\[
V(\tau) = 2A\tau^{2H},
\]
where the Holder exponent \( H \) must be in the range \( 0 < H < 1 \). The (time-dependent) diffusion coefficient \( D \) is then given by
\[
D = A \tau^{2H - 1},
\]
where \( A \) is the amplitude. The classical Brownian process is recovered for \( H = \frac{1}{2} \).

There are several important scaling properties of fractional Brownian motion (fBm). First, the distributions \( P(\Delta B(\tau)) \) of displacements for different time intervals collapse onto a single curve when normalized by \( \tau^H \). Second, the spectral density \( B_H(f) \) varies as \( f^{-\alpha} \), where \( \alpha = -(2H + 1) \). Finally, the correlations between sequential displacements are a function of \( H \). For the classical Brownian case \( (H = \frac{1}{2}) \), there are no correlations between successive displacements. However, for fBm with \( H \neq \frac{1}{2} \), there are correlations over infinite times. In particular, the normalized correlation function of a displacement \( \Delta B(\tau) \) with the past displacement \( \Delta B(-\tau) \) can be shown to be\(^{25,26}\)
\[
C(\tau) = \frac{\langle [\Delta B(\tau)][\Delta B(-\tau)] \rangle}{\langle [\Delta B(\tau)]^2 \rangle} = 2^{2H - 1} - 1.
\]
For \( H \neq \frac{1}{2} \), the normalized correlation is not zero and is \textit{independent} of the time interval \( \tau \). For \( H > 1/2 \), \( C(\tau) > 0 \), which indicates that a positive displacement is likely to be followed by a positive displacement, on the average. This phenomenon is termed as \textit{persistence}. Similarly, for \( H < \frac{1}{2} \), we have negative correlation or \textit{antipersistence}.

Fractional Brownian motion has not been previously used to describe trajectories of particles in fluid experiments. However, self-affine fractals have been used to describe the irregular interfaces that occur in several experiments, e.g., the immiscible displacement of one fluid by another during flow through porous media.\(^{27}\) A commonly used method\(^{28}\) of characterizing irregular curves \( y(x) \) is to compute their roughness \( w(L) \), defined as the rms value of the fluctuations of \( y(x) \) over a length scale \( L \):
\[
w(L) = \left\{ \langle [y(x) - \langle y(x) \rangle_L]^2 \rangle \right\}^{1/2},
\]
where \( \langle y(x) \rangle_L \) is the average of \( y(x) \) over a length scale \( L \). If \( w(L) \) scales as \( L^\beta \), then the curve is self-affine and \( \beta \) is said to be the "roughness exponent." It is expected that \( \beta = H \).

In this work, statistical properties of Brownian motion and its fractional generalization are used to characterize the observed particle motion on the fluid surface. However, the existence of this pseudorandom motion in a nearly steady wave field is in itself a striking result. We describe the experimental apparatus and the method used to make quantitative measurements of particle trajectories in Sec. II. The observations and their statistical analysis are presented in Sec. III, followed by a discussion of the results in Sec. IV.

II. DESCRIPTION OF THE EXPERIMENTS

A. Experimental apparatus and the basic flow

Experiments on particle transport in capillary waves were conducted in a square Plexiglas cell of dimensions 8 cm \( \times \) 8 cm \( \times \) 2 cm containing 1 cm of distilled water at 25°C. The cell is mounted on a precision electromagnetic shaker driven by a frequency synthesizer and power amplifier.\(^{29}\) An excitation frequency of 320 Hz yields waves oscillating at 160 Hz with a wavelength of 2.7 mm. The top of the cell is covered by a transparent Plexiglas plate with a 4 mm hole in the center to facilitate the addition of particles. Water was chosen as the working fluid rather than butyl alcohol, which was used previously.\(^{10}\) (Butyl alcohol is sensitive to surface contamination in a partially open cell, possibly because of the adsorption of water vapor. The wave patterns then change visibly over time. With water as the working fluid such difficulties are not encountered. Even a completely open cell produces patterns similar to a sealed one.) Poor
wetting by water and irregular contact lines at the side walls leads to some irreproducibility in the wave onset amplitude and inhomogeneity of the wave pattern near onset. Prewetting of the side walls before each experimental run helped to produce consistent wetting properties and repeatable values for the onset driving amplitude.

Measurements of the temporal and spatial characteristics of the wave pattern for water were made to allow comparison between the pattern characteristics and particle transport properties. The methods used to characterize the waves have been reported elsewhere. The surface pattern has square symmetry just above onset, and becomes disordered as a result of a transverse amplitude modulational (TAM) instability that has been studied theoretically by Milner. The early stages of this instability are visible in the shadowgraph image of Fig. 1(a). For water under the conditions of the present experiments, this instability is predicted to occur at $\epsilon \approx 0.0004$, where $\epsilon = (A - A_c)/A_c$ is the nondimensional driving amplitude ($A_c$ is the driving amplitude at the wave onset). This is sufficiently small that we could not resolve a purely unmodulated state with long-range order over lengths comparable to the size of the cell. On the other hand, for $\epsilon = 0.07$ [Fig. 1(a)], the time dependence is quite slow and the correlation length is about 6L. The pattern is completely disordered at higher amplitudes, as shown in Fig. 1(b). The variation of the characteristic frequency of the temporal motion and the spatial correlation length with $\epsilon$ is shown in Fig. 2; see Tufillaro et al. for a description of the methods used to obtain these parameters. The measurements show that there is a gradual loss of spatial order accompanied by an increasing fluctuation rate.

B. Measurement of particle trajectories

The particles used for studying the wave-induced transport are hollow ceramic spheres approximately 100–200 $\mu$m in diameter, much smaller than the wavelength of the surface pattern. The (mean) density of these spheres is approximately 0.7 g/cm$^3$ and hence the particles float on the surface of water. The particle is illuminated by a sheet of laser light parallel to the surface, and the scattered light is imaged by a video camera mounted above the cell. The observation proceeds until the particle encounters a side wall, where it is usually trapped. This imposes a practical upper limit to the length of the typical data records. To obtain better statistics, multiple sets of data (typically 5–10) are taken at each driv-

\[\text{FIG. 1. Shadowgraph images of the surface wave pattern at two different driving amplitudes. The image area (a small part of the cell) is } 4.2 \times 3.4 \text{ cm}^2. (a) Ordered (slightly time-dependent) wave pattern for nondimensional driving amplitude } \epsilon = 0.07; (b) \text{ Disordered state with more rapid fluctuations for } \epsilon = 0.35. \text{ The transition to disordered waves is mediated by a transverse amplitude modulational (TAM) instability that is visible in (a).} \]

\[\text{FIG. 2. (a) Variation of the characteristic pattern fluctuation frequency } f^* \text{ with driving amplitude } \epsilon. \text{ We estimate } f^* \text{ as the inverse of the autocorrelation time of the local image intensity. (b) Spatial correlation length } \xi \text{ (normalized by the wavelength } \lambda) \text{ as a function of } \epsilon. \text{ The correlation length declines with } \epsilon. \text{ The TAM instability arises at very small } \epsilon \text{ and its onset is unresolved for water at the driving frequencies of interest.} \]

ing amplitude $\epsilon$. In all, over a hundred data records were obtained at 20 different amplitudes. The longest record of the particle motion is 8 min, while the duration of the shortest record is about 80 sec at high driving amplitude. It is important to note that the statistical properties of the particle displacement, not the position, are of primary interest. We can obtain a sufficiently long set of particle displacements by combining those from several data sets for a given driving amplitude. This method proved useful in improving the statistical accuracy of certain measurements.

Digital processing of the video image provides the instantaneous location of the particle in the two horizontal coordinates parallel to the cell side walls at a rate of 5 Hz. (The vertical motion is only of order 0.2 mm and can be neglected.) The spatial resolution of the particle position is of the order of the pixel size, approximately 150 $\mu$m, which is also approximately equal to the particle size. The temporal resolution of 0.2 sec is sufficiently short to measure displacements of a few pixels. However, the digitization rate of 5 Hz is nearly two orders of magnitude smaller than the wave frequency, and hence it is not possible to obtain the detailed motion of the particle in each period of oscillation.

In order for the particle to follow the fluid motion well, the inertial force on the particle must be small compared to the forces because of viscosity and surface tension. For motion at the characteristic wave frequency, the inertial forces are clearly important. On the other hand, for motion at the slow time scale of the net particle drift, the inertial forces are much smaller. We estimate the Reynolds number based on particle diameter and particle speed to be less than unity and the surface tension forces to be several orders of magnitude larger than the inertial forces. However, since the surface tension forces are difficult to compute, an empirical test seemed desirable. Therefore we observed the motion of several particles simultaneously with the spreading of a spot of fluorescent dye (which mixes with the fluid near the surface). The particles follow nearby features of the dye pattern, which become very complex after a few seconds. Examples of the dye pattern for two different amplitudes are shown in Fig. 3.
Figure 4 shows particle trajectories for two values of \( \epsilon \). They are erratic and indicate the presence of particle (and fluid) transport on a scale much larger than the wavelength. Following Mandelbrot's notation, we denote the motion \( X(t) \) or \( Y(t) \) as a "record," and the trajectory \( \{X(t), Y(t)\} \) in the plane parallel to the surface as the "trail." The nature of the trajectories clearly depends on driving amplitude, as can be seen from the records shown in Fig. 5. It is quite evident that convolutions at smaller scales are more prominent in Figs. 5(c) and 5(d) than in Figs. 5(a) and 5(b).

III. ANALYSIS OF THE PARTICLE MOTION

A. Typical particle speed

Surprisingly, we observe long distance particle transport [Figs. 4(a) and 5(a) and 5(b)] even for small \( \epsilon \), where the wave pattern is largely ordered (though modulated) and the time dependence of the pattern is weak. Does the particle motion result from this weak time dependence of the pattern, or is it a finite-amplitude effect associated even with steady patterns? A qualitative test was made by making some measurements of the particle motion at lower driving frequency, where the pattern is nearly time independent; the complex motion was still present. Thus it appears that the time dependence of the pattern is not required to obtain long distance transport. In order to examine this issue quantitatively, we measured the typical particle speed \( v_p \) as a function of \( \epsilon \) over the range 0.04 < \( \epsilon \) < 0.4. (Since the waves do not fill the entire cell for \( \epsilon \) below approximately 0.04, it is not clear how the particle motion behaves for smaller \( \epsilon \) as we approach the quiescent state.) The typical particle speed was estimated as the root mean square of the fluctuations of the particle displacement in one second. We find that the particle speed (2–3 mm/sec) is a rather weakly increasing function of \( \epsilon \) as shown in Fig. 6(a), though the fluctuation rate \( f^* \) of the pattern changes by more than an order of magnitude over the same interval.

The instantaneous local fluid speed \( v_p \) resulting from the fast oscillation of the surface can be estimated as the product of the wave amplitude and the wave frequency. For the range of \( \epsilon \) studied here, \( v_p \) is in the range 60–100 mm/sec. Our observations imply that drift flows with speeds smaller than \( v_p \) by a factor of approximately 30 are superimposed on the wave field.

To make this discussion more quantitative, we show in Fig. 6(b) the ratio of the typical particle speed to the characteristic speed \( \lambda^* / 2 \) that one might expect if the pattern time dependence produced the transport. (The pattern typically moves locally by a distance \( \lambda / 2 \) in a time \( 1 / f^* \).) The ratio is slightly larger than 2 for \( \epsilon = 0.04 \), but decreases below unity as \( \epsilon \) is increased. This suggests that pattern variations are not the primary cause of particle transport at low \( \epsilon \), but that there might be a crossover to transport dominated by pattern variations at high \( \epsilon \). We also find that at a smaller driving frequency (100 Hz), where the pattern is nearly time independent, the characteristic ratio plotted in Fig. 6(b) is as high as 7 at small \( \epsilon \). We conclude from these tests that the small residual time dependence of the patterns at small \( \epsilon \) is not likely to be the primary cause of the particle transport for water waves.

Since it is not possible to completely eliminate the spatial modulations of the wave field for water as a result of the proximity of the secondary instability to the wave onset, we conducted some experiments on a second fluid, silicone oil (kinematic viscosity 0.10 cm²/sec), in which a regime with no spatial modulations could be observed near the wave onset. However, the ceramic particles do not remain on the surface of silicone oil. Therefore small dyed ethanol droplets (about 0.7 mm in diameter) were used instead, and their...
motion on the silicone oil was determined as already described. These droplets are larger than ideal, but were sufficient for semiquantitative observations. For silicone oil, significant random transport was found only above the onset of modulations. In the interval above the wave onset but below the onset of spatial modulations, we noted small particle drifts, but with a much smaller random component than was found for water waves.

B. Statistical properties

We have also studied the statistical properties of the particle displacements. Figures 7(a) and 7(b) show the particle displacement over a time interval of 1 sec, in coordinates parallel to the side walls for $\epsilon = 0.063$. The distribution of the displacements is shown in Fig. 7(c) and we find that the displacements in the two coordinates follow identical Gaussian distributions. Figure 8 confirms that similar behavior is observed even at high amplitudes (for $\epsilon = 0.375$), with the exception that the variance of the distribution is larger for the higher driving amplitude.

Next, we determine the variance $V(\tau)$ of the particle displacements as a function of the time interval $\tau$. The variance $V(\tau)$ is defined as $\langle (X(t + \tau) - X(t))^2 \rangle$, where $\langle \cdots \rangle$ denotes a time average over the record. Figure 9 shows the logarithmic variation of $V(\tau)$ for the records shown in Fig. 5. For the smaller value of $\epsilon$, $V(\tau)$ increases approximately as a power law with an exponent $2H \approx 1.2$ for $2 < \tau < 20$ sec, while for the larger driving amplitude, the variance increases almost linearly with the time interval ($2H \approx 1.0$). A more precise discussion of this power law behavior (and the range over which linearity might be expected) follows.

The roughness $w(\tau)$ of the record [Eq. (4)] also measures the rate of increase of displacements, but gives more precise estimates because it includes a greater degree of averaging. Examples of the variation of $w(\tau)$ with the time interval $\tau$ for the same two sets of data as for Fig. 9 are shown in Fig. 10. The roughness follows a power law with an exponent $\beta$ equal to $H$ (to within the accuracy of the measurements). In making these fits, we have taken care to exclude time intervals shorter than four times the autocorrelation time of the particle motion, because the apparent exponent is expected to be greater for correlated motion.31 We also cannot expect scaling at long time intervals, because the finite data records limit the degree of averaging.

![FIG. 7. (a) and (b)](image-url) Time variation of the particle displacements $\Delta X$ and $\Delta Y$ over an interval of one second for $\epsilon = 0.063$. (c) The probability distribution of the displacements. The two symbols represent the distributions of $\Delta X$ and $\Delta Y$. The solid curve is a Gaussian fit.

![FIG. 8. Same as for Fig. 7, but for $\epsilon = 0.375$; the variance is larger.](image-url)
The variance $V(\tau)$ as a function of the time interval $\tau$ for the records shown in Fig. 5. The variance increases approximately as a power law over a significant range of $\tau$, with an exponent $2H$. (a) $\epsilon = 0.063, 2H \approx 1.2$; (b) $\epsilon = 0.375, 2H \approx 1.0$.

In order to better judge the extent to which a power law adequately describes the data, we show in Fig. 11 the slope of $w(\tau)$, smoothed over about one-third decade. Though there is clearly some variation of the slope with $\tau$, it is reasonably constant over at least one decade, and is significantly larger than 0.5 for the lower $\epsilon$. Figure 12 shows the variation of the exponent $\beta (= H)$ with the driving amplitude $\epsilon$. For small $\epsilon$, we find that $\beta \approx 0.7$, and it decreases gradually with $\epsilon$, asymptotically approaching 0.5 (corresponding to the classical Brownian motion) at large $\epsilon$.

To confirm that the particle displacements along the two coordinate axes are statistically independent, we compute the distribution of the magnitude $\Delta R(\tau) \equiv [\Delta X(\tau)^2 + \Delta Y(\tau)^2]^{1/2}$ of the particle displacement. If the displacements along the two orthogonal directions are independent Gaussian distributions with same variance, then the distribution of $\Delta R(\tau)$ must follow a Rayleigh distribution. Figure 13(a) shows the variation of $\Delta R(\tau)$ for a
time interval of 1 sec for $\varepsilon = 0.063$. Figure 13(b) demonstrates that the Rayleigh distribution fits the data reasonably well. We obtain similar results for higher driving amplitudes (Fig. 14).

These measurements are consistent with the hypothesis that the particle motion can be characterized as a two-dimensional fractional Brownian trail whose characteristic exponent varies with the driving amplitude. In order to further test this hypothesis, we need to determine whether the particle motion displays the various scaling properties of fBm. As mentioned in the Introduction, one of the characteristics of fBm is that the distributions of the particle displacements over various time intervals can be scaled. Figure 15 shows that the distribution of the normalized displacements $\tau^{-H} [\Delta X(\tau)]$ for various time intervals; it is clear that the probability distributions of the normalized displacements scale as expected. If the same graph is prepared with $H = \frac{1}{2}$, the scaling is inferior. Similar behavior was found at all driving amplitudes.

We have also obtained the spectral density of the record. Figure 16 shows the Fourier spectra averaged over several records, for two values of the driving amplitude. The exponent $\alpha$ from power law fits to the spectra are given in Fig. 17 for various $\varepsilon$, along with the predicted value $[\sim 2H - 1]$ for fBm. However, the data are only marginally precise enough to exclude a constant slope of $-2$, independent of $\varepsilon$.

Perhaps the most striking predicted property of fBm with $H > \frac{1}{2}$ is the persistence (lack of statistical independence) of the motion. As discussed in the Introduction, for fBm with $H > \frac{1}{2}$, the predicted correlation $C(\tau)$ between the past and future displacements should be positive and independent of the time interval $\tau$. The predicted value for $\varepsilon = 0.04$ (corresponding to the largest measured value of $H$) is equal to $0.26 \pm 0.07$. Figure 18 shows the experimental measurements of $C(\tau)$ for two values of $\varepsilon$. It is clearly not constant. There is generally an initial fast decline with a de-
cay time of several seconds, followed by a slower (and irregular) decay to zero over a much longer interval, roughly 20 sec at $\varepsilon = 0.04$. The fast decay occurs in a time similar to the decay of the ordinary autocorrelation function of the record.

The particle motion must of course have large short term correlations, since the velocity is continuous. But after the fast decay, does $C(t)$ remain positive or not? The data seem weakly consistent with $C(\tau) = 0$ at long times, in disagreement with the fBm hypothesis and the measured $H > \frac{1}{2}$. On the other hand, the finite length of the data records leads to poor statistical accuracy in determining long term correlations. The uncertainty in the calculation of $C(\tau)$ for large $\tau$ is estimated to be at least $\pm 0.1$, making it difficult to determine reliably whether any residual correlation of the expected magnitude (0.2) remains at long times. For large driving amplitudes, as shown in Fig. 18 (b), $C(\tau)$ decays rather convincingly to zero, but of course this is expected since $H = \frac{1}{2}$.

To fully characterize the particle transport process, one must know not only the exponent $H$, but also the amplitude $A$, which itself is a function of $\varepsilon$. Since $V(\tau) = 2 A \tau^{2H}$, the amplitude $A$ is equal to half the variance $V(\tau)$ when the time interval $\tau$ is 1 sec. The variation of $A$ with $\varepsilon$ is shown in Fig. 19. The measurements suggest a gradual but very modest increase in $A$ with $\varepsilon$. We know that $A$ approaches zero for very small $\varepsilon$, but quantitative experiments could not be done in that regime.

IV. DISCUSSION

We have investigated the behavior of the particle motion on a water surface undergoing capillary wave motion. A surprising result is that there is irregular particle motion even at low $\varepsilon$, where the pattern is ordered, though modulated in space. The drift motion is weaker than the local instantaneous wave speed by more than an order of magnitude. The evidence indicates that the weak time dependence of the pattern itself is probably not the primary cause for the particle motion at small $\varepsilon$ for water waves, though pattern varia-

FIG. 16. Power spectra (arbitrary units) of the records of the particle motion and corresponding power law fits $P(f) \sim f^{-\alpha}$: (a) $\varepsilon = 0.063$, $\alpha = -2.14$; (b) $\varepsilon = 0.375$, $\alpha = -2.05$.

FIG. 17. Variation of the exponent $\alpha$ of $P(f)$ with $\varepsilon$. The measured exponents are compared with the expected value $\left( -2H - 1 \right)$, computed from Fig. 12, drawn as a smooth curve. The error bars indicate one standard deviation. The data marginally exclude a constant exponent of $-2$.

FIG. 18. Variation of the correlation $C(\tau)$ between past and future displacements with time interval $\tau$: (a) $\varepsilon = 0.04$; an initial fast decline with a decay time of several seconds is followed by a slower (and irregular) decay to zero over roughly 20 sec. (b) $\varepsilon = 0.375$; the decorrelation of the past and future displacements occurs over only a few seconds.
Spatial (and temporal) modulations showed very
independent) have been suggested to lie arbitrarily

The particle displacements in the two horizontal
coordinates are independent Gaussian processes. For
small driving amplitudes, the variance of the particle displacements in­
creases as a power law with an exponent

The particle displacements in the two horizontal coordinates are independent Gaussian processes. For small driving amplitudes, the variance of the particle displacements increases as a power law with an exponent $H > 1/2$. It varies only weakly with $\epsilon$, except for very small $\epsilon$, where quantitative measurements are not possible. We do not show the units of $A$ because they depend on $H$.

Experiments on silicone oil (Sec. III) below the onset of spatial (and temporal) modulations showed very little irregular particle motion. Spatially modulated states (even if time independent) have been suggested to lie arbitrarily close in parameter space to solutions with complex spatial structure. Though the implications for transport are unclear, it seems possible that spatial modulations are a prerequisite for the irregular particle transport we have observed. The experiments on both fluids are consistent with this possibility.

The particle displacements in the two horizontal coordinates are independent Gaussian processes. For small driving amplitudes, the variance of the particle displacements increases as a power law with an exponent $2H$ larger than that for normal Brownian motion. We determine $H$ indirectly but more accurately through the roughness exponent $\beta$ of the records. It decreases with $\epsilon$, asymptotically approaching unity at large $\epsilon$. The distributions of the displacements at different time intervals collapse onto a single curve when scaled by $\tau^{-1/2}$. While these observations and the power spectra suggest that the motion may be characterized as a fractional Brownian process, measurements of persistence (future/past correlations) at long times were inconclusive as a result of the limited length of the records. In any case, it is clear that the particle motion is not well described as an ordinary Brownian process for small $\epsilon$, even over times long compared to the autocorrelation time of the motion. We were unable to locate other quantitative measurements of particle trajectories (in wave fields or other hydrodynamic flows) with which to compare this result.

In a related investigation, we have conducted experiments on tracer dispersion by capillary waves. The conclusions of that study are consistent with those reached in the present paper. Studies of particle trajectories may also be useful in characterizing low dimensional chaotic flows, where an exponent $H < 1/2$ is an indicator of trapping of particles in subdomains of the flow. In turbulent flows, changes in the statistical behavior of particle trajectories may provide a sensitive method of detecting instabilities.

Note added in proof: The authors recently became aware of a possible framework for understanding the drift flows in the case of strong excitation [Fig. 1(b)]. Herterich and Has­selmann proposed that tracer diffusion in surface waves can be understood in terms of random fluctuations of the Stokes drift, which is quadratic in the wave amplitude. Though the mean drift velocity is zero for standing waves, individual particles "will experience drift-velocity fluctuations relative to the mean value due to the statistical fluctuations of the local wave amplitudes in a random sea." An estimate indicates that these fluctuations are of the correct order of magnitude to explain our observations; a quantitative test is in progress.

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7 For examples, see M. Van Dyke, An Album of Fluid Motion (Parabolic, Stanford, CA, 1982).
13 See, for instance, H. Lamb, Hydrodynamics (Dover, New York, 1932).
23 S. Jones and W. Young, submitted to J. Fluid Mech.
33 While this manuscript was in preparation, we learned that a similar conclusion has been reached independently by M. Rabinovich (private communication).
34 J.-P. Eckmann and I. Procaccia, to appear in *Nonlinearity*. 