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Optical Heterodyne Study of the Taylor Instability in a Rotating Fluid*

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The Taylor instability in a rotating fluid confined between two cylinders has been investigated by light scattering. For rotation rates \( f \) asymptotically close to the critical rotation rate \( f_c \), we find that the amplitude of the ordered flow varies as \( (f - f_c)^{0.5+0.0} \). The axial structure of the ordered flow is sinusoidal near \( f_c \), but harmonics become substantial for \( f - f_c \) large. The main features of the Landau approach to hydrodynamic instabilities are thus confirmed.

A number of recent papers\(^1\)\(^-\)\(^8\) have discussed the behavior of fluids near hydrodynamic instabilities. One motivation for this work is the suggestive analogy between fluid instabilities and second-order phase transitions, both of which exhibit an order parameter that grows from zero in the neighborhood of a critical point. Another is the hope of understanding the transition to turbulence, which occurs by means of a succession of progressively more complicated instabilities. Most of the previous work has been concerned with the Rayleigh-Bénard or convective instability. We have performed an experimental study of the Taylor instability\(^7\) in a rotating fluid, and find excellent agreement with the Landau picture\(^6\)\(^-\)\(^8\) of hydrodynamic instabilities.

The Taylor instability occurs when a fluid is confined between an outer stationary cylinder and an inner rotating one. If the rotation rate \( f \) exceeds a critical value \( f_c \), the radial pressure gradient and the viscous forces are not sufficient to provide the required centripetal acceleration of the fluid, and a new flow pattern perturbs the \( z \)-independent Couette flow. Superimposed on the original azimuthal flow \( V_\phi(r) \), there is now (Fig. 1) a toroidal roll pattern much like that of the Rayleigh-Bénard instability. Near \( f_c \), the velocity components \( V_r \) and \( V_\phi \) are of course quite

\[ V_r \sim (f - f_c)^{0.5+0.0} \]

\[ V_\phi \sim (f - f_c)^{0.5+0.0} \]

FIG. 1. (a) Side view of the cylindrical apparatus and light paths. The flow pattern shown schematically is actually superimposed on a much faster azimuthal flow perpendicular to the page. The axis of the rotating cylinder is along the \( z \) direction. (b) Top view showing the method used to mix the scattered and reference beams. Each arm is 30 cm long.
small. If \( f \) is increased considerably beyond \( f_c \),
there are further instabilities in which the vortices acquire
 circumferential waves, and eventually a transition to a turbulent state (nonperiodic
in time) occurs. The first instability is the subject of this paper.

According to the Landau theory and subsequent work one expects
that just above \( f_c \), the radial velocity should be of the form
\( V_r = A_1(r, \epsilon) \cos(\rho k_z z) \),
where \( \epsilon = (f - f_c)/f_c \) and the amplitude \( A_1 \) varies
as \( \epsilon^{1/2} \). Fluctuations may influence this dependence
for \( \epsilon < 10^{-5} \), but this region is probably
not experimentally accessible. As \( \epsilon \) is increased
higher harmonics should appear as a result of the
nonlinear terms in the Navier-Stokes equations, so that

\[
V_r = \sum_{\rho} A_\rho(r, \epsilon) \cos(\rho k_z z).
\]

The discussion above refers to the steady-state situation. The response time of the system
when perturbed from equilibrium is expected to
diverge as \( \epsilon \to 0 + \). The major new result of the
present work is the measurement of \( A_p(\epsilon) \) for \( p = 1, 2 \).

In order to study \( V_r(r, z, \epsilon) \) we have utilized
laser light (5 mW at 5145 Å) scattered at an angle
of 171° from the forward direction by a dilute sus-
pen sion of 2-µm polystyrene latex spheres in wa-
ter, as shown in Fig. 1. The scattered light was
mixed with an unscattered but attenuated beam by
use of a Michelson-like interferometer, and the
power spectrum of the photocurrent was ob-
tained from a real-time spectrum analyzer. The
power spectrum exhibits a peak approximately
200 Hz wide at a frequency in the range 0 to
10 000 Hz. For our geometry the ratio of the lo-
cal radial velocity to the mean frequency of the
peak is \( 1.94 \times 10^{-5} \) cm sec\(^{-1}\) Hz\(^{-1}\). The scattering
volume was experimentally determined to be
about 1 mm long in the radial direction and 0.2
mm in the orthogonal directions.

The fluid was contained between an inner black
aluminum cylinder of radius 1.555 cm, and a pre-
cision-bore Pyrex tube of inner radius 2.540 cm.
The cylindrical region was 30 cm long, and was
 temperature controlled to within 0.03°C. Tem-
perature control was necessary because \( f_c \) changes
by \( 2\% \) per degree because of the temperature de-
pendence of the kinematic viscosity of the fluid.
At 27.0°C, we found \( f_c \) to be 0.0561 ± 0.0002 Hz,
which is consistent with the predictions of Chan-
drasekhar. (A quantitative comparison is not
possible because the linear stability theory has
only been evaluated for special geometries in

which the ratio \( R_1/R_2 \) of radii either is \( 0.5 \) or
approaches unity. However, previous work has
established the correctness of theoretical predic-
tions for \( f_c \) and \( k_r \).) The rotation rate was mea-
sured electronically to an accuracy of 0.02% and
was constant to within 0.04%.

The \( z \) dependence of \( V_r \) was studied halfway be-
tween the inner and outer cylinders by translat-
ing the apparatus with the optics unchanged. In
Fig. 2 we present the results for \( \epsilon = 0.014 \). The maximum value of \( V_r \) is only 0.06\( V_r \),
so that the periodic structure is a fairly small perturbation
on the azimuthal flow. As predicted for small \( \epsilon \),
the flow is nearly sinusoidal. However, there is
a small second-harmonic term, which can be de-
tected by the difference in magnitude of the posi-
tive and negative peaks. The amplitudes of the
fundamental and second-harmonic terms (ob-
tained by computer Fourier analysis) are \( A_1 \)
= 0.0155 cm/sec and \( A_2 = 0.0019 \) cm/sec, and the
solid curve shows that these two terms alone pro-
duce an excellent fit, with a fundamental wave
number \( k_1 = 3.05 \times 10^{-1} \). The product \( k_1 R_2 \) is 7.75,
which can be roughly compared to Chandrasek-
har's prediction \( k_1 R_2 = 6.4 \) for the case \( R_1/R_2 = 0.5 \).

As \( \epsilon \) becomes larger, the second and higher
harmonics increase in importance relative to the
fundamental term, because of their dependence
on higher powers of \( \epsilon \). In Fig. 3, we present \( V_r(z) \)
for \( \epsilon = 0.465 \). Here the magnitude of \( V_r \) has a
higher and sharper peak in the outward flowing
regions than in the inward flowing ones. We find
that three terms are necessary to produce a fit
to within the accuracy of the experiment, and the
amplitudes are \( A_1 = 0.1030 \) cm/sec, \( A_2 = 0.0356 \)
cm/sec, and \( A_3 = 0.0072 \) cm/sec. The wave num-
ber \( k_1 \) is 3.20 cm\(^{-1}\), and is independent of \( \epsilon \) over
the range 0<ε<0.5 to within the precision of the measurements (5%). The precision is limited by small thermally induced drifts in the flow pattern near ε=0.

The dependence of the Fourier coefficients Aₙ on ε is shown in Fig. 4. Points marked by a cross were obtained by fixing the rotation rate, allowing the system to equilibrate (which required at least 15 min when ε<0.01), measuring \( V_r(x) \), and then performing a complete Fourier analysis. The first two amplitudes \( A_1 \) and \( A_2 \) can also be obtained from a simpler procedure, in which only the magnitudes \( P_+ \) and \( P_- \) of the positive and negative peaks in \( V_r(x) \) at each value of ε need be measured. We utilize the combinations \( \frac{1}{2}(P_+ + P_-) = A_1 + A_3 + \ldots \) and \( \frac{1}{2}(P_+ - P_-) = A_2 + A_4 + \ldots \) to accomplish this. The amplitude \( A_2 \) is at most 7% of \( A_1 \) in the range of interest, and can be removed as a small correction by using \( A_2(ε) \) as obtained from Fig. 4 and other data. Removal of \( A_4 \) is unnecessary, as it was found to be negligible over the range of ε studied. Points for which \( A_1 \) and \( A_2 \) were obtained in this manner are indicated by dots in Fig. 4. These points are consistent with those obtained by complete Fourier analysis of \( V_r(z) \).

To test the prediction that \( A_1 \) varies as \( ε^{1/2} \), a least-squares analysis was performed, in which the weighted sum of the squares of the deviations from \( D(\epsilon - f_c)^{1/2} \) was minimized with \( D \), \( \beta \), and (if desired) \( f_c \) as free parameters. The weighting was determined by assuming the measurement error \( ΔA_1 \) to be independent of ε. We determined experimentally that 0.0649 < \( f_c < 0.0653 \) Hz by noting the rotation rate at which the photocurrent power spectrum no longer has a peak at \( ν > 0 \).

The uncertainty arises partly from the fact that frequency shifts of less than 200 Hz are not detectable, and partly from what appear to be slight variations in \( f_c \) from run to run. We find that \( A_1 = (0.145 \pm 0.013)e^{0.003 \pm 0.004} \) cm/sec, where the uncertainties are due mainly to the uncertainty in \( f_c \).

If we treat \( f_c \) as a free parameter, the best fit occurs when \( f_c = 0.0549 \) Hz and \( \beta = 0.52 \). We conclude that the measurements are consistent with the prediction \( A_1 \sim ε^{1/2} \). The data of Berge and Dubois,⁴ who obtained \( \beta > \frac{1}{2} \) for the convective instability, may have been influenced by the second and higher harmonics, which were not measured in their experiments.

Applying the same approach to \( A_2(ε) \), we found that \( A_2 = (0.063 \pm 0.005)e^{0.003 \pm 0.004} \) cm/sec represents the data quite well. In order to make sure that we were actually observing asymptotic behavior, we repeated the analysis after eliminating points for \( ε > 0.1 \), and found the exponent to be 0.76 ± 0.06, unchanged except for a larger error. We have not considered \( A_4 \) in detail because the data are at present insufficient for an accurate determination of the \( ε \) dependence of \( A_4 \).

Our results confirm the Landau approach to fluid behavior asymptotically near an instability leading to a spatially periodic but time-independent flow. In particular, a single spatial Fourier component dominates near the critical point, and its amplitude grows as \( ε^{1/2} \). However, the exponent 0.77±0.03 of the second-harmonic term, which becomes important further from the critical point, is not understood at the present time. Hopefully this and other experiments will stimulate theoretical work aimed at understanding the nonasymptotic region.

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Breaking the Roton Barrier: An Experimental Study of Motion Faster than the Landau Critical Velocity for Roton Creation in He II†

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We report the first observation of objects moving through He II with equilibrium drift velocities \( \vec{v} \) beyond the Landau critical velocity \( v_c \) for roton creation. With \( T \sim 0.4 \) K, \( P \sim 2 \) bar, \( F \sim 2 \) kV cm\(^{-1} \), \( \vec{v} \sim v_c \) for negative ions is larger than a theoretical prediction by a factor of \( \sim 10^5 \). The vortex-ring nucleation rate is found to decrease with \( F \) above 300 V cm\(^{-1} \), thus resolving apparent inconsistencies between earlier experiments.

In his celebrated explanation of superfluidity, Landau\(^ 1 \) showed that the kinetic energy of a liquid flowing at velocity \( v \) through a tube (or that of a heavy object moving through the liquid) cannot be dissipated through the creation of an excitation of energy \( \epsilon \) and momentum \( p \) in the liquid unless \( v \gg \epsilon/p \). For He II the minimum value of \( \epsilon/p \) is nonzero, occurring close to the roton minimum in the elementary excitation spectrum, so that a critical velocity \( v_c = (\epsilon/p)_{\text{min}} \approx 50 \) m sec\(^{-1} \) exists, below which dissipation ought not to occur in the superfluid. Measured critical velocities are usually orders of magnitude smaller, because of the onset of vortex formation at lower velocities, but Rayfield\(^ 2 \) reported that the drift velocity \( \vec{v} \) of negative ions in He II under pressure \( P \geq 12 \) bar below 0.6 K appeared to reach and to be limited by \( v_c \) when the applied field was raised to about 70 V cm\(^{-1} \). This has remained the only known situation to which Landau's original criterion for the breakdown of superfluidity appears to be relevant.

Takken\(^ 3 \) has considered roton creation by negative ions moving at velocities slightly greater than \( v_c \) on the basis of a wave radiation model in which each ion is assumed to generate a conical wave of coherent roton radiation, much like the disturbance produced by an airplane breaking the sound barrier. By analogy with the aerodynamic case, a rapid increase in drag is expected as the velocity increases past \( v_c \). Takken concluded that an upper bound on \( v \) is given by \( v_{\text{db}} = v_c (1 + 10^{-12} F^2) \), where the electric field \( F \) is in V cm\(^{-1} \). A \( 1\% \) increase of \( v \) above \( v_c \) would therefore require \( F > 10^3 \) V cm\(^{-1} \), implying that any increase of \( v \) beyond \( v_c \) ought to be almost impossible to observe experimentally.

An attempt by Neep\(^ 5 \) and Meyer\(^ 6 \) to test this remarkable assertion was thwarted by an unexpected increase in the vortex nucleation rate \( \nu \) with falling temperature, such that at 0.3 K only vortex rings, and no bare ions, arrived at their collector. Our recent observation\(^ 7 \) that the field emission current at 0.3 K in He II increases dramatically with \( P \) above 12 bar, and is temperature independent below 0.4 K, seemed to be inconsistent with Neep and Meyer's result. Approximate values of \( \nu \) deduced\(^ 8 \) from the field-emission measurements apparently indicated the feasibility of our present experiment to test Tak-